Primary User Enters the Game: Performance of Dynamic Spectrum Leasing in Cognitive Radio Networks

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Abstract—Dynamic spectrum leasing (DSL) is one of the schemes proposed for dynamic spectrum sharing (DSS) in cognitive radio networks. In DSL, spectrum owners, denoted as primary users, dynamically adjust the amount of secondary interference they are willing to tolerate in response to the demand from secondary transmitters. In this correspondence we investigate how much can be gained by primary users if this limited interaction with secondary system is allowed, compared to a scheme in which the interference cap allowed by primary users is fixed *a priori* by a regulatory authority. To that end, we define performance metrics for both primary and secondary systems based on the theoretically achievable multiuser sum-rate of the secondary system and analyze both schemes' behavior with respect to different system parameters. This analysis shows that (i) in dynamic environments DSL based schemes may present an important advantage over other schemes with fixed interference constraints, and (ii) DSL schemes are robust against inaccurate *a priori* information that may degrade system performance.

Index Terms—Cognitive radio, DSL, dynamic spectrum sharing, dynamic spectrum leasing, game theory, multiuser decoding.

I. INTRODUCTION

RECENT studies [1], [2] have showed that the paradox of apparent scarcity of radio spectrum while most of the bands are underutilized occurs mainly due to the inefficiency of traditional static spectrum allocation policies. This has prompted proposals for various *dynamic spectrum sharing* (DSS) approaches, such as *dynamic spectrum leasing* (DSL) [3]–[5].

As opposed to passive spectrum sharing by the primary users considered in many previous DSS proposals, leasing, as proposed in [3]–[5], means that the primary users have an incentive (e.g. monetary rewards as leasing payments) to allow secondary users to access their licensed spectrum. Therefore, the primary user plays an active role in interference management and dynamically controls how much interference

must be allowed from the secondary system. In DSL the primary user is assumed to adapt its interference cap (IC), denoted by $Q_0 \in [0, \bar{Q}_0]$, which is the maximum total interference the primary user is willing to tolerate from secondary transmissions at any given time.

While in [4], [5] we proposed a game theoretical framework in order to model and analyze a practical DSL scheme, in this work we are interested in the best performance achievable by a general DSL scheme. Hence, we investigate the performance improvement that can be expected by a DSL based paradigm with respect to passive spectrum sharing schemes which do not allow dynamic primary-secondary network interaction based on proactive primary systems. The proposed analysis results into a Stackelberg game formulation of the interactions between primary and secondary systems.

Stackelberg games have been previously used to model cognitive radio systems. A cooperative scheme in which secondary users actively collaborate with the primary user transmissions was proposed in [6]. However a high degree of awareness and global synchronization is required by the two in principle heterogeneous systems. A Stackelberg game formulation is also used in [7], [8] to describe the high level interactions within the network. However these works do not consider physical layer issues such as modulation used or impact of the primary / secondary interference in the attainable rates of the system.

As opposed to previous works, in the present correspondence we use performance metrics based on the multiuser sum-rate attainable by the secondary system. We choose this performance metric because it is a fundamental limit against which practical schemes can be compared, while it is independent of particular DSL implementations. Most information theoretic work on the cognitive radio channel assumes a certain amount of knowledge by the secondary system on the transmitted primary codeword that allows the use of dirty paper coding by the secondary system (see i.e. [9]. In this work, however, we relax this assumption and treat primary transmission purely as noise[1].

The rest of this paper is organized as follows: In Section II we introduce a signal model and the assumed decoding strategies employed by primary and secondary users. Next, in Section III we propose a general family of performance metrics for both primary and secondary users and quantitatively show the performance gain of a DSL based scheme

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[1]In practical systems strict causality hinders the *a priori* knowledge of the primary user's codeword at secondary transmitters. On the other hand, at secondary receivers primary signals cannot be reliably decoded due to SNR considerations or synchronization issues.

over schemes in which the amount of interference tolerated by the primary system is fixed. A practical example with two concrete performance metric functions is presented in Section IV. Finally, Section V concludes this paper.

II. SYSTEM AND SIGNAL MODEL

In this work, for simplicity of exposition, we assume uncorrelated block fading channels and the existence of a single primary link and a single secondary receiver of interest[2]. K secondary transmitters are interested in accessing this spectrum band to the maximum possible extent. While Matched filter (MF) decoding is a popular decoding structure due to its simplicity and performs reasonably well in systems with weak cross-channels, in an interference limited regime it is clearly suboptimal and it is outperformed by joint decoding of multiple users. We will consider here the optimal joint maximum likelihood multiuser decoder (ML MUD) [10]. Note that multiple schemes exist today for the practical implementation of multiuser decoding, such as successive interference cancellation (hard) or multiuser turbo decoding (soft).

We require a limited awareness by the primary receiver of the secondary system. Therefore secondary transmissions are considered as noise in the primary decoding process. On the other hand, the base station of the secondary system is assumed to have a MUD for the K secondary transmissions while the primary signal is assumed to be undecodable and thus treated as noise.

A. Signal model

The primary user is denoted as user 0 while the secondary transmitters are labeled as users 1 through K. A discrete-time representation of the received signals at the primary and secondary receivers can be written as

$$r_p[n] = h_{p0}s_0[n] + \sum_{k=1}^{K} h_{pk}[n]\tilde{s}_k[n] + \sigma_p n_p[n]; \quad (1)$$

$$r_s[l] = h_{s0}\tilde{s}_0[l] + \sum_{k=1}^{K} h_{sk}[l]s_k[l] + \sigma_s n_s[l] \quad (2)$$

where n and l represent the discrete sampling times at primary and secondary receivers respectively, h_{pk} and h_{sk} are the effective channels from k-th transmitter to the primary and secondary receivers respectively. If $s_k(t)$ denotes the signal transmitted by the k-th user, then $s_k[n]$ denotes a synchronously sampled and $\tilde{s}_k[n]$ an asynchronously sampled version of $s_k(t)$. Finally $n_p[n]$ and $n_s[n]$ are iid Gaussian processes normalized to have variance 1 so that σ_p^2 and σ_s^2 represent the noise power levels at the primary and secondary receivers, respectively.

We denote the transmit power of the k-th user as $p_k \doteq E\{\|s_k[n]\|^2\} \doteq E\{\|\tilde{s}_k[n]\|^2\}$ for $k = 0, 1, \ldots, K$. Note that this assumes that any deviations on the received power due to front-end and bandwidth differences are absorbed into the effective channel coefficients. Then it is straightforward to see

[2]While in the schemes presented in [4], [5] more complex set-ups were employed, here we consider a simplified scenario to better illustrate the theoretical advantage of DSL over schemes that employ fixed interference cap levels.
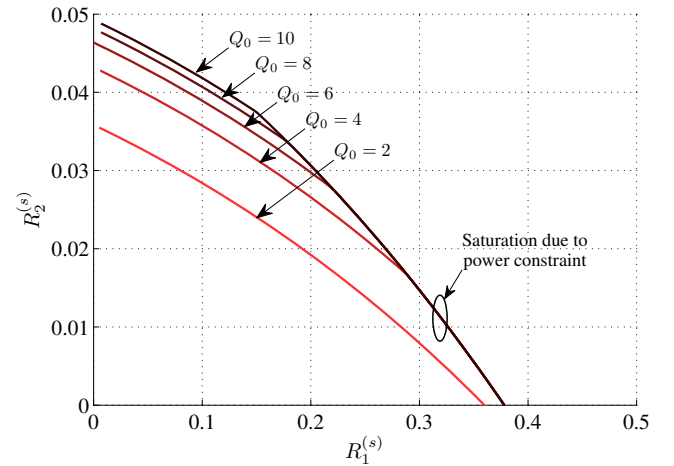

Fig. 1. Secondary system 2-user rate region for different values of Q_0.

that the actual interference power generated by the secondary system at the primary user is given by

$$I_0 \doteq \sum_{k=1}^{K} |h_{pk}|^2 p_k. \quad (3)$$

B. Decoding strategy

Primary user. The maximum achievable rate per channel use assuming secondary interference as noise at the primary system is given by

$$R_p \leq W_p \log \left(1 + \frac{|h_{p0}|^2 p_0}{I_0 + \sigma_p^2}\right) \quad (4)$$

where W_p represents the bandwidth employed by primary transmissions and the transmitted power p_0 is determined by the required quality of service (QoS) and the interference cap selected.

Secondary user. If optimal multiuser decoding is used in the secondary system with bandwidth W_s, the maximum achievable sum rate at the secondary receiver treating primary transmissions as noise is, see e.g. Sec 15.3.6 in [11],

$$R_s < W_s \log \left(1 + \frac{\sum_{k=1}^{K} |h_{sk}|^2 p_k}{|h_{s0}|^2 p_0 + \sigma_s^2}\right) \quad (5)$$

for each of the allowed secondary power assignments p_k with $k = 1, \ldots, K$, which are determined by the maximum interference allowed at the primary user $I_0 \leq Q_0$ and secondary user individual power constraints $p_k < \bar{p}_k$.

The rate region obtained with this scheme is similar to the one obtained in a Gaussian Multiple Access Channel, with the peculiarity that on top of having individual power constraints, secondary users have a weighted global power constraint. The individual rates achieved by each secondary user will depend on the particular coding/decoding strategy used.

From the constraint $I_0 \leq Q_0$ and given the definition of I_0 we have that the term $\sum_{k=1}^{K} |h_{sk}|^2 p_k$ in (5) is upper bounded by a monotonically increasing affine function of Q_0. Then it is apparent from (5) that while the upper bound on the secondary sum-rate is monotonically increasing with Q_0, the growth rate decreases with Q_0 due to the logarithmic relation

with $\sum_{k=1}^{K} |h_{sk}|^2 p_k$. Figure 1 shows an example of the rate region obtained in a two user secondary system where the channel from user 1 to primary is much weaker than the one from user 2 for different values of Q_0. While in general the region is increasing with Q_0, the effect of the individual power constraints of the secondary nodes translates into the partial saturation of the achievable rate region.

III. PERFORMANCE COMPARISON

Although performance evaluation of cognitive radio systems is important in comparing and ranking different paradigms, it has received only a limited attention in current literature [12]. Even for the relatively simple model considered in this work, there exist several possible evaluation metrics: maximum achievable sum-rates at primary and secondary systems R_p and R_s respectively, power dissipated by a given user p_k, interference generated at the primary user I_0, probability of primary outage $\mathrm{prob}\{I_0 > Q_0\}$, fairness among users, and spectral efficiency, among others. Therefore, an adequate utility function must first be defined in order to compare DSL based paradigms with other schemes.

A. Performance metric

While in our model a natural performance metric for the secondary system should be an increasing function of the attained sum rate R_s, the primary user's utility needs further considerations. Since the primary user suffers from a (permitted) interference I_0 from the secondary system, in order to maintain its QoS the primary user transmitted power p_0 is increased with respect to an exclusive use of the frequency band ($I_0 = 0$). We denote this increment in the transmitted power by Δp_0. Hence the primary user needs an incentive to allow secondary users to use its managed spectrum. We assume here that the secondary system compensates the primary user with a payment (monetary or of other nature) related to the generated interference I_0. As a result, the utility functions for primary and secondary systems can be written as:

$$U_p = u_p(I_0, \Delta p_0), \qquad (6)$$
$$U_s = u_s(R_s, I_0) \qquad (7)$$

where primary utility $u_p(\cdot)$ is growing with I_0 and decreasing with Δp_0, while secondary utility $u_s(\cdot)$ grows with R_s. We additionally assume that when the interference constraint is violated, that is when $I_0 > Q_0$, the penalization imposed by the primary system to the secondary system implies $U_p = \infty$, $U_s = -\infty$. This penalty discourages the secondary system from violating the allowed interference cap.

B. Performance gain

For a given interference cap Q_0 the secondary utility U_s is maximized for the secondary power vector $\boldsymbol{p} \doteq [p_1 p_2 \cdots p_K]^T$ provided that

$$\boldsymbol{p}^\star(Q_0) = \arg\max_{\boldsymbol{p}}\{u_s(R_s(\boldsymbol{p}, Q_0), I_0(\boldsymbol{p}))\} \qquad (8)$$
$$\text{subject to} \quad I_0(\boldsymbol{p}) \leq Q_0, \ \boldsymbol{p} \leq \bar{\boldsymbol{p}}$$

where we defined $\bar{\boldsymbol{p}} \doteq [\bar{p}_1 \bar{p}_2 \cdots \bar{p}_K]^T$ and the operator $\leq$ denotes element by element comparison. Here we have

explicitly shown the dependence of R_s on Q_0. We define the corresponding primary and secondary utilities as $U_p^\star(Q_0) \doteq U_p(\boldsymbol{p}^\star(Q_0), Q_0)$ and $U_s^\star(Q_0) \doteq U_s(\boldsymbol{p}^\star(Q_0), Q_0)$, respectively.

If the primary user fixes *a priori* the interference cap Q_0 in a time varying environment its expected utility is given by $E[U_p^\star(Q_0)]$ where the expectation is taken with respect to the channel realizations. On the other hand, in a DSL scheme we allow the primary system to dynamically adjust the allowed interference cap Q_0. We can now compute the maximum achievable utility for both types of schemes:

Schemes with fixed Q_0: If the primary user chooses the value of Q_0 that maximizes the expected utility and uses it for all channel realizations, its utility is given by

$$\bar{U}_p^{\text{fixed}} = \max_{Q_0}\{E[U_p^\star(Q_0)]\}. \qquad (9)$$

DSL schemes: On the other hand, in a DSL-based system, the primary will choose the interference cap Q_0 to maximize its own utility for each channel realization. The best expected primary utility achievable in this dynamic environment is

$$\bar{U}_p^{\text{dsl}} = E[\max_{Q_0}\{U_p^\star(Q_0)\}]. \qquad (10)$$

It is easy to see from (9) and (10) that $\bar{U}_p^{\text{dsl}} \geq \bar{U}_p^{\text{fixed}}$, with equality if and only if the optimal Q_0 is constant for all channel realizations. In the next section we will use a simple example to show that indeed the gain obtained by a DSL scheme can be significant.

Remark: In deriving (10) we implicitly formulated the interaction between the primary and secondary systems as a Stackelberg game [13], in which the primary user acts as Stackelberg *leader* and the secondary system acts as *follower*. While this is a natural model for cognitive radio systems in which the primary can always act unilaterally while secondary users have to adapt their actions to the imposed constraint [14], practical implementations that achieve this behavior are a topic of further research.

C. Practical considerations

While in the previous analysis we did not discuss how a practical scheme could achieve the derived performance, we present here some practical issues that need to be taken into account. In the proposed Stackelberg game we assumed that both primary and secondary systems have perfect knowledge of all system parameters, and thus they can optimize their performance by maximizing their own utilities.

However, even for classical schemes with fixed interference cap it is difficult for a secondary system to determine how much interference it causes to a primary receiver. A practical implementation would require secondary users to estimate their channels to the primary receivers. This could be performed in duplex primary systems with reciprocal uplink and downlink channels by monitoring the primary signal levels.

On the other hand, DSL based schemes could achieve the operating point predicted by the Stackelberg equilibrium without requiring full knowledge of the system parameters. For example, if the utilities are such that the Stackelberg equilibrium coincides with the unique Nash equilibrium of the

game, it can be achieved via an iterative game between the primary and secondary systems. The practical DSL scheme proposed in [4] only requires the primary system to broadcast the values of I_0 and Q_0. Even though this approach requires a certain degree of awareness about the secondary network by the primary system, it results in a practical scheme that can easily be implemented in practice.

Due to space limitations, in the following we will focus on theoretically achievable performance and disregard practical implementation considerations.

IV. EXAMPLE

For illustration purposes, in this section we assume that the utilities associated with primary and secondary users are respectively

$$U_p = I_0 - \mu_P \Delta p_0, \qquad (11)$$
$$U_s = \mu_R R_s - I_0 \qquad (12)$$

with the additional restriction that $I_0 \leq Q_0$. That is, the primary system obtains a reward proportional to the suffered interference I_0 through the corresponding charge to the secondary system. Without loss of generality we assume here the payoff per unit of interference equal to 1. The primary user has a cost associated to the extra power Δp_0 required to maintain its desired QoS, priced at the rate of μ_P. The reward for the secondary system is proportional to the achievable sum rate R_s priced at the rate of μ_R. Note that whereas these utilities keep the spirit of (6) and (7), they are also simple enough to obtain analytical results.

A. Analysis

Assuming equality in (5) we may rewrite (12) as

$$U_s = \mu_R W_s \log \left(1 + \frac{\sum_{k=1}^{K} \frac{|h_{sk}|^2}{|h_{pk}|^2} \tilde{p}_k}{\sigma_s^2 + |h_{s0}|^2 p_0} \right) - \sum_{k=1}^{K} \tilde{p}_k \qquad (13)$$

where we have defined $\tilde{p}_k \doteq |h_{pk}|^2 p_k > 0$.

In order to maximize U_s with respect to $\tilde{p}_k$ we first note that for fixed $\sum_{k=1}^{K} \tilde{p}_k = I_0$, U_s is growing with respect to a convex combination of the (positive) ratios $|h_{sk}|^2/(I_0|h_{pk}|^2)$. Hence, for a fixed I_0, U_s is maximized when all the allowed secondary interference I_0 is allocated to the secondary transmitters with the largest ratios $|h_{sk}|^2/|h_{pk}|^2$ up to their individual power constraints. Formally, if we define the indexes of the sorted effective channels as $\{i_1, i_2, \ldots, i_K\}$ such that

$$\frac{|h_{si_1}|^2}{|h_{pi_1}|^2} \geq \frac{|h_{si_2}|^2}{|h_{pi_2}|^2} \geq \cdots \geq \frac{|h_{si_K}|^2}{|h_{pi_K}|^2}, \qquad (14)$$

the optimal power assignment is given by

$$\tilde{p}_{i_k}^\star \doteq \begin{cases} |h_{pi_k}|^2 \bar{p}_{i_k}, & \delta_k < I_0, \\ I_0 - \delta_{k-1}, & \delta_{k-1} \leq I_0 \leq \delta_k, \\ 0, & \text{elsewhere}, \end{cases} \qquad (15)$$

where we defined $\delta_k \doteq \sum_{l=1}^{k} |h_{pi_l}|^2 \bar{p}_{i_l}$. Then we may define the instantaneous channel ratio η as

$$\eta \doteq \frac{\sum_{k=1}^{K} \frac{|h_{sk}|^2}{|h_{pk}|^2} \tilde{p}_k^\star}{I_0}. \qquad (16)$$

Note that when the secondary individual power constraints are not active $I_0 \leq |h_{pi_1}|^2 \bar{p}_{i_1}$, hence η reduces to the largest channel ratio pair: $\eta = \max_k\{|h_{sk}|^2/|h_{pk}|^2\}$. Otherwise η is a convex combination of the strongest channel ratio pairs.

Remark: While the simple utility (12) leads to an opportunistic access scheme that does not take into account fairness among secondary users, in the general setting U_s could take a more complex form in order to guarantee fairness. However this analysis lies out of the scope of the present work.

Using (16) and substituting (15) in (13) we have that

$$U_s = \mu_R W_s \log \left(1 + \frac{\eta I_0}{\sigma_s^2 + |h_{s0}|^2 p_0} \right) - I_0. \qquad (17)$$

Equating the derivative of (17) with respect to I_0 to zero, we obtain the global U_s maximizer. Taking into account the additional constraint $I_0 < Q_0$, one obtains that the optimal I_0 is given by

$$I_0^\star(Q_0) = \min \left(Q_0, W_s \mu_R + (\sigma_s^2 + |h_{s0}|^2 p_0)/\eta \right). \qquad (18)$$

As in [4], we will assume here that $p_0 = \bar{\gamma}(Q_0 + \sigma_p^2)/|h_{p0}|^2$ where $\bar{\gamma}$ is the target primary SINR to assure a required QoS. Then it follows that

$$U_p^\star(Q_0) = I_0^\star - \mu_P \Delta p_0 \qquad \text{and} \qquad (19)$$
$$U_s^\star(Q_0) = \mu_R W_s \log \left(1 + \frac{\eta I_0^\star}{\sigma_s^2 + |h_{s0}|^2 p_0} \right) - I_0^\star, \qquad (20)$$

where we omitted the dependence of $I_0^\star$ on Q_0.

The maximal primary utility is achieved by a DSL system maximizing $U_p^\star(Q_0)$. Given the restriction $I_0 \leq Q_0$ and since $U_p^\star$ is growing with $I_0^\star$ and decreasing with Q_0 it can be shown that $U_p^\star(Q_0)$ is maximized when $I_0^\star = Q_0$. Hence the optimal instantaneous Q_0 is given by

$$Q_0^\star = \frac{\eta W_s \mu_R - \frac{|h_{s0}|^2}{|h_{p0}|^2} \bar{\gamma} \sigma_p^2 - \sigma_s^2}{\eta + \frac{|h_{s0}|^2}{|h_{p0}|^2} \bar{\gamma}}. \qquad (21)$$

Note that, as can be seen from (19) above, the optimal strategy for the primary user is heavily dependent on the scenario and thus cannot be fixed a priori. In order to compute the expected gain in a dynamic environment for a DSL based scheme over a paradigm with fixed Q_0, given by $\bar{U}_p^{\text{dsl}} - \bar{U}_p^{\text{fixed}}$, we further need to define a channel model and compute the average of (19) with respect to all channel realizations. Although, in general, the expected gain cannot be computed in a closed form, it can easily be evaluated numerically for any given set of parameters.

B. Numerical results

We assume that channels to secondary receiver h_{pk} and h_{sk} are Rayleigh distributed with $E\{|h_{pk}|^2\} = E\{|h_{sk}|^2\} = 1$. The remaining system parameters are $K = 3$, $\bar{p}_k = 100$, $\bar{Q}_0 = 10$, $\sigma_p^2 = \sigma_s^2 = 1$ while the transmitters are considered fixed with $|h_{p0}|^2 = |h_{s0}|^2 = 1$. We employ normalized bandwidth $W_p = W_s = 1$, target SINR $\bar{\gamma} = 1$ and resource prizes initially set to $\mu_P = 0.1$ and $\mu_R = 2$.

Figure 2 shows the comparison between a DSL based scheme and a scheme in which the allowed interference cap Q_0 is fixed for the given set of system parameters. In Fig. 2(a)
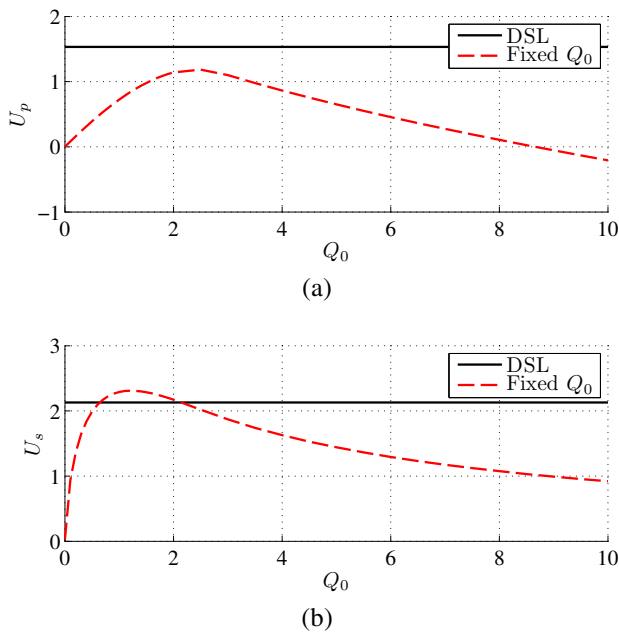
Fig. 2. Primary/secondary users average performance in a time varying environment. (a) Primary user performance. (b) Secondary user performance.

we can see that even if a fixed system were to use the optimal $Q_0 \approx 2.5$, the primary utility attainable by a DSL based scheme is about 25% larger than the one of the fixed scheme. On the other hand, if we look at the secondary utility obtained by a DSL based scheme compared to a scheme with fixed Q_0, as shown in Fig. 2(b), we can see that while fixed schemes perform better than DSL for a small range of Q_0 values, for the optimal operating point of the fixed scheme ($Q_0 \approx 2.5$) DSL performs slightly better than the fixed scheme. That is, in this setting both primary and secondary users can benefit from the use of a DSL scheme. Moreover in a DSL based scheme the allowed interference at the primary is computed on line, and thus it does not need to be fixed *a priori*. Hence, DSL schemes can be robust against inaccurate knowledge of the system parameters that may degrade both primary and secondary performance at the expense of the extra complexity required for dynamically setting the value of Q_0. Note from Fig. 2 that a small change in the Q_0 value for the fixed scheme can significantly degrade the global system performance.

However, as we pointed out above the advantage of DSL based schemes vanishes if the optimal primary user action Q_0 is independent from the channel realization. If we assume high reward for the secondary system sum rate, that is $\mu_R = 100$, the best responses for both primary and secondary users turn to be $I_0 = Q_0 = \bar{Q}_0$, not depending on the channel realization. In this case DSL and fixed schemes with $Q_0 = \bar{Q}_0$ turn out to be equivalent achieving $U_p \approx 8.99$ and $U_s \approx 235$.

V. CONCLUSIONS

In this correspondence we analyzed the performance gain that a primary user can expect by allowing a limited interaction with secondary systems. For a family of performance metrics based on the secondary user sum-rate we showed that DSL based schemes can outperform classical schemes in dynamic environments. Moreover, since the allowed interference at the primary is computed on-line, DSL schemes are robust to inaccuracies on the *a priori* knowledge on system parameters that can degrade both primary and secondary performances.

These results advocate the design of practical dynamic spectrum sharing schemes based on DSL type architectures with a limited interaction between primary and secondary systems, as opposed to previously proposed fixed interference cap frameworks.

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