Dynamic Spectrum Leasing in Cognitive Radio Networks via Primary-Secondary User Power Control Games

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***Abstract*—Hierarchical dynamic spectrum access (DSA) has received the most attention in recent years as the solution for better spectrum utilization. In this paper, on the other hand, we develop a framework for dynamic spectrum leasing (DSL). Power control in hierarchical DSA networks only involves that of controlling secondary user transmissions. Thus, in game theoretic formulations of power control in cognitive DSA networks only secondary users are considered as players of the game. In proposed dynamic spectrum leasing, on the other hand, the primary users are rewarded for allowing secondary users to operate in their licensed spectrum. Thus, in the proposed DSL networks the primary users have an incentive to allow secondary users to access the spectrum whenever possible to the maximum extent. We develop a game theoretic framework for such dynamic spectrum leasing in which primary users actively participate in a non-cooperative game with secondary users by selecting an interference cap on the total interference they willing to tolerate. We establish that the proposed primary-secondary user power control game has a unique Nash equilibrium. Performance of a DSL system based on the proposed game model is compared through simulations under different linear receivers at the secondary base station.**

***Index Terms*—Cognitive radios, dynamic spectrum access, dynamic spectrum leasing, dynamic spectrum sharing, game theory, power control.**

I. Introduction

THERE has been a growing consensus in recent years that the scarcity of radio spectrum is mainly due to the inefficiency of traditional *fixed* spectrum allocation policies [1], [2]. As a result, there are three possible *dynamic* spectrum access (DSA) approaches that have been floated as possible solutions to improve spectrum utilization: a.) open-sharing, b.) hierarchical-access, and c.) dynamic exclusive use [1], [3]. While open-sharing advocates a model similar to the highly successful concept of industrial, science and medicine (ISM) bands, the second option of hierarchical spectrum access essentially allows improving spectrum utilization in current spectrum allocations. As a result, hierarchical access in which secondary users are allowed to opportunistically access the spectrum on the basis of no-interference to the primary (licensed) users, is arguably the method that has received the most attention in recent literature. In particular, various spectrum underlay and overlay methods that have been proposed and investigated in recent years are aimed at achieving hierarchical DSA. In DSA, there is a primary system that owns the spectrum rights and the secondary users are expected to access the spectrum only when primary users do not use their spectrum, and on the basis of non-interference to the primary users. The burden of interference avoidance/management in sharing the spectrum is thus mainly placed on the secondary transmitters. This has naturally led to cognitive radios as an enabling platform in realizing such dynamic spectrum sharing since these units have built-in cognition that can be used to observe, learn from and adjust to the RF interference.

Cognitive radios [4], that can be defined as smart radios with built in cognition, are especially suited for dynamic spectrum access due to their ability to observe and assess their RF surroundings and learn from and orient to their environment. Secondary cognitive transmitters may access a spectrum band that is already licensed to another user (called the primary user) as long as it can properly adjust its transmission parameters, in particular the transmit power, so as not to interfere with and interrupt the primary transmissions. Thus, power control is an important issue in this spectrum sharing process. In [5] and [6], authors have proposed schemes for power control among such secondary cognitive radios. However, power control in cognitive hierarchical-DSA networks only involved that of controlling secondary user transmissions. Thus, for example, in game theoretic formulations of power control in DSA networks, the primary users are not considered as decision makers, i.e. they do not actively participate in the spectrum sharing process. This is so, because in hierarchical dynamic spectrum sharing framework there is no incentive for a primary user to do anything to facilitate (or hinder) secondary transmissions. In a recent paper [7] considered both power and rate control via a game theoretical approach. Again only secondary users were considered as active players of the game. In [8] primary users are allowed to choose a certain transmission rate, however still not as a direct participant of the same non-cooperating game of secondary users. Thus, these schemes are essentially similar to the power control schemes in traditional wireless networks [9].

Many researchers have used game theoretical methods to

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analyze the problem of resource allocation in wireless networks. For example, in [10] the authors proposed a non-cooperating power control game based on a specific energy efficient utility function that is common to all users. They established that their proposed game has a unique Nash Equilibrium (NE). In [11], by realizing the NE in the game in [10] may not be optimum, the authors further introduced the concept of Pareto efficiency into the game. They imposed a linear pricing function to gain better overall performance. This energy efficient game was later generalized to linear minimum mean-squared-error receivers (LMMSE) in [12], and showed that the modified game also converges to a unique NE. In [13], the authors generalized this game further by considering quality-of-service (QoS) constraints. A recent summary on game theoretical approaches used for energy efficient resource allocation in wireless networks can be found in [9].

In this paper, on the other hand, we consider the option of *dynamic spectrum leasing* (DSL) as an approach for better spectrum utilization. Spectrum leasing is one of the solutions that has been suggested under the third option of dynamic exclusive-use model in which the spectrum licensees are also granted the rights to sell or trade their spectrum to third parties [1], [3]. As opposed to passive spectrum sharing by the primary users as in hierarchical-DSA, leasing would mean that the primary users have an incentive (e.g. monetary rewards as leasing payments) to allow secondary users to operate in their licensed spectrum. In particular, we have proposed the concept of dynamic spectrum leasing in which primary users dynamically adjust the extent to which they are willing to lease their spectrum. In this paper, we have also developed a possible game theoretic framework to achieve such dynamic spectrum leasing in a cognitive radio network. As mentioned above, in game theoretic formulations of power control in cognitive hierarchical-DSA networks only the secondary users are considered to be the players of the game. The primary users' influence is ignored beyond that of causing passive interference. However, in proposed dynamic spectrum leasing networks the primary users do have an incentive to allow secondary users to access the spectrum whenever possible to the maximum extent since they will be rewarded (e.g. monetarily) for allowing secondary users to operate. Thus, in our game theoretic framework for dynamic spectrum leasing the primary users are also incorporated into the player set of the game. In the proposed formulation, primary users actively participate in a non-cooperative game with secondary users by selecting a reasonable interference cap (IC) on the total interference they are willing to tolerate. They are rewarded for sharing their licensed spectrum, but are penalized if they do not meet their own target QoS. Simultaneously, the secondary users aim to achieve energy efficient transmissions, while not causing excessive interference to the primary users. We establish the existence of a unique Nash equilibrium in the proposed game for dynamic spectrum leasing.

The performance of cognitive radio systems based on the proposed game theoretical dynamic spectrum leasing approach is studied with different linear detectors at the secondary receiver. In particular, it is observed that with the matched-filter (MF) receiver, far before total secondary user interference exceeds the maximum allowed interference cap, secondary users will be transmitting at their maximum allowed transmit power while still not achieving their target signal-to-interference-plus-noise ratio (SINR) requirement. On the other hand, it is shown that the linear minimum mean-squared-error receiver is able to support more secondary users to achieve their target SINR (compared to that with the MF receiver) while still keeping the total secondary user interference under the interference cap set by the primary user. This, of course is due to the superior interference suppression capability of the LMMSE receiver.

Other methods for power control in cognitive radios, besides those based on game theory, have also been investigated, for example, in [14], [15] (and references therein). In particular, a joint power control and beam-forming via either weighted least squares or admission control was recently studied in [16]. An opportunistic power adaptive method for secondary users was proposed in [17]. This scheme seems to relax the synchronization and perfect channel state information requirements, which might be an advantage in the presence of fading.

The remainder of this paper is organized as follows: Section II presents the proposed system and game models for dynamic spectrum leasing. Section III analyzes the proposed primary-secondary user power control game to establish the existence of a unique Nash Equilibrium (NE) with linear receivers. Section IV investigates the performance of a dynamic spectrum leasing network based on the proposed game theoretical scheme through numerical simulations, and discusses the performance comparison between the LMMSE receiver and the MF receiver. Section V concludes the paper with a discussion on future research directions.

II. System and Game Models

We propose a cognitive dynamic spectrum leasing wireless network architecture in which the system that owns the spectrum property rights (called the primary system) willingly and actively attempts to share its spectrum with transmitters from secondary systems. Without loss of any generality, in this paper we assume one primary system that owns the spectrum rights and only one secondary system that is aiming to access this spectrum whenever it is feasible. It is to be noted that spectrum leasing is a suggested alternative by the FCC to better improve spectrum utilization under the spectrum property rights granted in dynamic exclusive-use model [1]. The idea is that the primary system has the freedom to lease its spectrum bands to secondary transmitters. Obviously leasing would mean that the secondary system will have to pay a certain compensation to the primary system for this spectrum access, and naturally the amount of compensation can expected to be proportional to the amount of allowed spectrum leasing by the primary system. Thus, as opposed to hierarchical DSA based systems, the primary system in our proposed dynamic spectrum leasing network has an incentive to allow secondary user transmissions to the maximum possible extent whenever it is affordable.

For simplicity, in this paper we assume that the primary and secondary systems consist of, respectively, only one primary user and K secondary users as shown in Fig. 1. Note that,

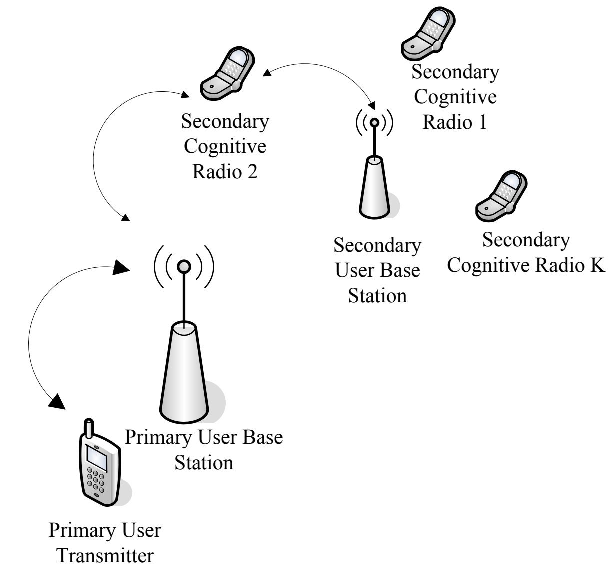

Fig. 1. The primary-secondary user communications system model.

although we limit ourselves to one primary transmitter for the simplicity of exposition, the proposed scheme can be extended to include more than one primary user. Since the key ingredient in the proposed concept of dynamic spectrum leasing is the interaction between the primary system and the secondary system, a single primary user is enough to demonstrate the key aspects of dynamic spectrum leasing while avoiding extraneous complications. However, in a realistic network, of course, there will be multiple primary transmitters and their own internal interactions will add an important dimension to the problem of dynamic spectrum leasing. There is one primary receiver and one common secondary receiver in the system (again,generalization to more than one is possible). The cross correlation coefficients between the signalling waveforms of the k-th secondary user and that of a primary user is denoted by ρ_{kp}, between a primary user and the k-th secondary user is by ρ_{pk} and between the k-th and the j-th secondary users is by ρ_{jk} for all $k, j \in \{1, \cdots, K\}$. For simplicity, throughout we will assume that $\rho_{kp} = \rho_{pk} = \rho_{sp}$, for all $k \in \{1, \cdots, K\}$. The channel gain between the k-th secondary user and the common secondary receiver is h_{sk}, between the k-th secondary user and the primary receiver is h_{pk}, between the primary user and the primary receiver is h_{p0}, and between the primary user and the common secondary receiver is h_{s0}.

In the proposed formulation, the primary user can adapt its interference cap, denoted by Q_0, which is the maximum total interference the primary user is willing to tolerate from all secondary transmissions. However, the primary user should always first strive to achieve its target SINR to ensure its required QoS. This is an important constraint in the concept of dynamic spectrum leasing since it is expected that the primary system should first focus on its communication needs and spectrum leasing is only an option to improve the spectrum utilization. Note that, the QoS requirement in conjunction with the chosen interference cap will directly determine the primary user's transmit power level. By adjusting the interference cap, the primary user can indirectly control the total transmit power the secondary users impose on the channel at any given time. All secondary users adapt their transmission powers to achieve a certain transmission quality. However, their transmission powers must be carefully controlled in order to ensure low interference to the primary user (within the allowed interference cap) as well as to other secondary users. We use P_0 and p_k to represent transmission powers of the primary user and the k-th secondary user, respectively.

In the above cognitive dynamic spectrum leasing network, the primary and secondary users interact with each other by adjusting their actions in response to those of the others: the primary user by adjusting its interference cap (which, in turn, determines its transmit power) and the secondary users by controlling their transmit power levels. In essence, both primary as well as secondary users act as rational decision makers, thereby making game theory a natural framework to analyze and predict the behavior of this system. Formally, we model our proposed scheme as the following non-cooperative game:

1) Players: $\mathcal{K} = \{0, 1, 2, ..., K\}$, where 0-th user is taken to be the primary user and $k = 1, 2, ..., K$ represents the k-th secondary user.
2) Action space: $\mathcal{P} = \mathcal{Q} \times \mathcal{P}_1 \times \mathcal{P}_2 ... \times \mathcal{P}_K$, where $\mathcal{Q} = [0, \bar{Q}_0]$ represents the primary user's action set and $\mathcal{P}_k = [0, \bar{P}_k]$, for $k = 1, 2, ..., K$, represents the k-th secondary user's action set. $\bar{Q}_0$ and $\bar{P}_k$ represent, respectively, the maximum allowed interference cap of the primary user and the maximum allowed transmission power of the k-th secondary user. The action vector of all users is denoted by $\mathbf{p} = [Q_0, p_1, ..., p_K]$, where $p_k \in \mathcal{P}_k$ and $Q_0 \in \mathcal{Q}$. The action vector excluding the action of the k-th user, for $k = 0, 1, 2, ..., K$, is customarily denoted by $\mathbf{p}_{-k}$.
3) Utility function: We use $u_k(p_k, \mathbf{p}_{-k})$ $, \forall k = 1, 2, ..., K$ to represent the k-th secondary user's utility function and $u_0(Q_0, \mathbf{p}_{-0})$ to represent the primary user's utility function.

Throughout this paper, we assume that the primary receiver is based on a matched-filter detector since we are limiting ourselves to a primary system with only a single user. However, it is possible to modify the proposed scheme for situations in which the primary receiver can be an advanced multiuser detector, as will be required when one considers a primary system with multiple transmitters. Assuming a matched-filter based primary receiver, the primary user's target SINR is defined as:

$$\bar{\gamma}_0 = \frac{h_{p0}^2 P_0}{Q_0 + \sigma^2}, \tag{1}$$

where P_0 and Q_0 represent the primary user's transmission power and its chosen interference cap, respectively, and σ^2 is the variance of the additive noise at the primary receiver. Note that, since Q_0 is the maximum interference from secondary users the primary user is willing to tolerate at any given time, $\bar{\gamma}_0$ in (1) represents the worst-case transmission quality the primary user can expect with its chosen Q_0. Since this worst-case SINR needs to guarantee a required QoS constraint, the primary user's transmit power is thus directly determined by its chosen interference cap Q_0[1]. On the other hand, the primary user's actual received SINR is given by,

$$\begin{aligned}\gamma_0^{(P)} &= \frac{h_{p0}^2 P_0}{\sum_{j=1}^K h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2} = \frac{h_{p0}^2 P_0}{I_0 + \sigma^2} \\ &= \frac{\bar{\gamma}_0 Q_0}{\sum_{j=1}^K h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2} + \frac{\bar{\gamma}_0 \sigma^2}{\sum_{j=1}^K h_{pj}^2 \rho_{sp}^2 p_j + \sigma^2}\end{aligned}$$

where we have denoted the total interference from all secondary users to the primary user by $I_0 = \sum_{j=1}^K h_{pj}^2 \rho_{sp}^2 p_j$. Thus, as long as $I_0 \leq Q_0$, the primary user will meet its target SINR requirement of $\bar{\gamma}_0$. Motivated by above discussion, we propose the following primary user utility function [18]:

$$\begin{aligned}u_0(Q_0, \mathbf{p}_{-0}) = Q_0 &- \mu_1 \left[(Q_0 - I_0)^2 u(Q_0 - I_0) \right] \\ &- \mu_2 \left[\left(e^{(I_0 - Q_0)} - 1 \right) u(I_0 - Q_0) \right], \quad (2)\end{aligned}$$

where $u(.)$ is the step function with $u(x) = 1$ for $x \geq 0$ and $u(x) = 0$ for $x < 0$, and μ_1 and μ_2 are positive pricing coefficients. Note that the pricing functions (the second and the third terms in (2)) are introduced to ensure that the primary user's required QoS is not be undermined. When the primary user's instantaneous SINR is less than the target SINR, i.e. when $Q_0 < I_0$, the primary user is significantly penalized because it doesn't achieve its required transmission quality. On the other hand, when its instantaneous SINR is greater than the target SINR, i.e. $Q_0 > I_0$, the primary user is still relatively penalized. This is because when the primary user achieves its target SINR, it does not need to transmit at too high a power wasting its own power as well as causing more interference to all other users operating in the same portion of the spectrum. In other words, when the primary user sets an interference cap, the shared spectrum should be fully utilized. i.e. the total interference from the secondary users should be as close as possible to that interference cap.

The goal of secondary user's in this system is to achieve the most energy efficient transmissions. Hence, we use the following commonly used utility function that reflects the energy efficiency in transmissions in wireless networks as the secondary user utility function [11], [12]:

$$u_k(p_k, \mathbf{p}_{-k}) = \frac{R_k f\left(\gamma_k^{(s)}\right)}{p_k}, \quad (3)$$

where R_k is the transmission rate of the k-th secondary user, $f\left(\gamma_k^{(s)}\right) = \left(1 - e^{\left(-0.5\gamma_k^{(s)}\right)}\right)^M$ is the efficiency function, $\gamma_k^{(s)}$ and is the k-th secondary user's SINR, and M is the number of bits in one packet. Essentially, (3) defines the secondary user utility as the number of successfully transmitted bits per unit transmission power. When a particular secondary user is able to transmit at the power level p_k that maximizes this utility, that user is said to achieve the most energy efficient transmission.

[1] For this reason, we may call this primary-secondary user game a power control game, although the basic action of the primary user is in setting its interference cap Q_0.

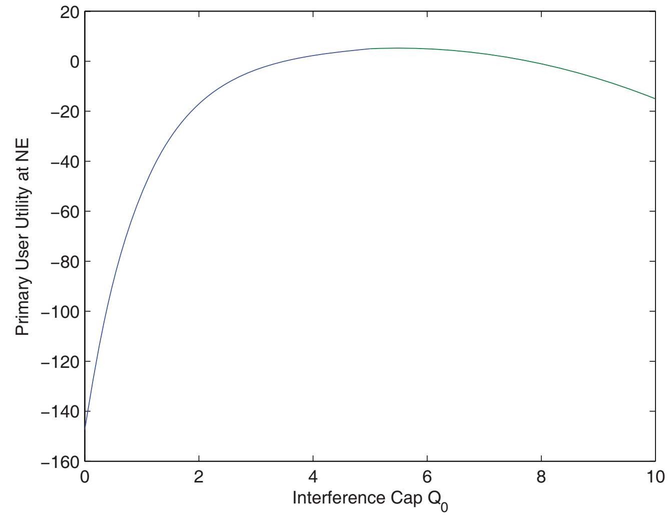

Fig. 2. Concavity of the primary user's utility function. $I_0 = 5$.

III. EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM OF THE PROPOSED PRIMARY-SECONDARY USER GAME WITH LINEAR RECEIVERS

A. The Power Control Game with the MF Secondary Receiver

First, we assume that the secondary system employs an MF receiver. Then the k-th secondary user's SINR, $\forall k = 1, 2, \cdots, K$, at the common secondary receiver output is:

$$\begin{aligned}\gamma_k^{(s)} &= \frac{h_{sk}^2 p_k}{\sum_{j \neq k} h_{sj}^2 p_j \rho_{j,k}^2 + \sigma^2 + h_{s0}^2 \rho_{ps}^2 P_0} \\ &= \frac{h_{sk}^2 p_k}{\sum_{j \neq k} h_{sj}^2 p_j \rho_{j,k}^2 + \frac{h_{s0}^2 \rho_{ps}^2 \bar{\gamma}_0 Q_0}{h_{p0}^2} + \sigma^2 \left(1 + \frac{h_{s0}^2 \rho_{ps}^2 \bar{\gamma}_0}{h_{p0}^2}\right)}. \quad (4)\end{aligned}$$

Note that, from (4) it can easily be seen that $\frac{\partial \gamma_k^{(s)}}{\partial p_k} = \frac{\gamma_k^{(s)}}{p_k}$.

1) Existence of a Nash Equilibrium: A Nash equilibrium exists in game $G = (K, \mathcal{P}, u_k(.))$, if for all $k = 0, 1, 2, \cdots, K$, the k-th user's action set, $\mathcal{P}_k$, is a nonempty convex, and compact subset of some Euclidean space $\mathbb{R}^N$, and $u_k(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k [11]. Note that, here $\mathcal{P}_0 = \mathcal{Q}$ and $p_0 = Q_0$ for the primary user.

The power action sets of the primary user and the secondary users are closed subsets of $\mathbb{R}$. Furthermore, it's easy to check that the utility functions of the primary user and the secondary users are continuous in $\mathbf{p}$. Finally, since quasi-concavity of the utility function of the secondary users have been proved in [11], we only need to show the quasi- concavity and the continuity of the utility function of the primary user. Clearly, u_0 is continuous in $\mathbf{p}$. Furthermore, when $0 \leq Q_0 \leq I_0$, the primary user's utility function reduces to $u_0 = Q_0 + \mu_2(1 - e^{-(Q_0 - I_0)})$. The second order derivative is $u_0'' = -\mu_2 e^{-(Q_0 - I_0)} < 0$. Thus, it is concave in Q_0. On the other hand, when $I_0 \leq Q_0$, the second order derivative of the primary user's utility function is $u_0'' = -2\mu_1 < 0$. Thus as can be seen in Fig. 2 the utility function is again concave in Q_0.

Therefore, the utility functions of both primary and secondary users satisfy all the required conditions, so that there exists at least one NE in this game. In the following, we show that, in fact, this NE is unique.

2) Uniqueness of the NE: Before establishing the uniqueness of the NE in the proposed power-control game we need the following definition. The best-response function of player k is $r_k^*(\mathbf{p}_{-k}) = \{p_k \in \mathcal{P}_k : u_k(p_k, \mathbf{p}_{-k}) \geq u_k(p_k', \mathbf{p}_{-k})$ for all $p_k' \in \mathcal{P}_k\}$. If we let $\mathbf{r}(\mathbf{p}) = (r_1^*(\mathbf{p}_{-1}), \cdots, r_K^*(\mathbf{p}_{-K}))^T$, then $\mathbf{r}(\mathbf{p})$ is termed the best-response correspondence of the game. Note that, if the best-response function has a fixed point $\mathbf{p} = \mathbf{r}(\mathbf{p})$, then clearly it is a NE of the game.

Our interest in the best-response function of a game is due to the following result the has been established in [19]: If the best-response correspondences of the primary and the secondary users are *standard functions*, then the NE in this game will be unique.

A function $\mathbf{r}(\mathbf{p})$ is said to be a standard function if it satisfies following three properties [19]:

1) *Positivity:* $\mathbf{r}(\mathbf{p}) > 0$.
2) *Monotonicity:* If $\mathbf{p} \geq \mathbf{p}'$, then $\mathbf{r}(\mathbf{p}) \geq \mathbf{r}(\mathbf{p}')$.
3) *Scalability*: For all $\mu > 1$, $\mu\mathbf{r}(\mathbf{p}) \geq \mathbf{r}(\mu\mathbf{p})$.

The best-response correspondence of the secondary users in the proposed primary-secondary user power-control game can be obtained by setting $u_k'(p_k, \mathbf{p}_{-k}) = 0$, for $k = 1, \cdots, K$, which leads to $f'(\gamma_k^{(s)})\gamma_k^{(s)} - f(\gamma_k^{(s)}) = 0$. If we denote by $\gamma_k^{(s)} = \gamma^*$ the solution to above equation, γ^* is determined only by function $f(.)$. Since we have assumed that all secondary users have the same efficiency function $f(.)$, this implies that the SINR corresponding to the best-response is the same for all secondary users: i.e. $\gamma_k^{(s)} = \gamma^*$ is the same for all secondary users. Hence, the best-response of the k-th secondary user is the following transmit power which provides it with the optimal SINR γ^*:

$$r_k^*(\mathbf{p}_{-k}) = \frac{1}{h_{sk}^2}\gamma^*\left(\sum_{j\neq k} h_{sj}^2 p_j \rho_{j,k}^2 + \sigma^2(1 + \frac{h_{s0}^2\rho_{ps}^2\bar{\gamma}_0}{h_{p0}^2}) + \frac{h_{s0}^2\rho_{ps}^2\bar{\gamma}_0 Q_0}{h_{p0}^2}\right). \tag{5}$$

Note that, $r_k^*(\mathbf{p}_{-k})$ can be shown to be a standard function for $\forall k = 1, \cdots, K$ by following an approach similar to that was given in [20]. Taking into account the finite upper bound of the secondary user's action set $\bar{P}_k$, the secondary user's best-response correspondence is $r_k^*(\mathbf{p}_{-k}) = \min\{\bar{P}_k, p_k^*\}$, where p_k^* is the k-th secondary user's transmission power which provides it with the optimum SINR γ^*. When some of the secondary users cannot achieve γ^*, they will transmit at their maximum possible transmit power $\bar{P}_k$. In this case, the NE is still unique [11].

In showing that the best-response function of the primary user is standard, we first establish that the best-response correspondence of the primary user utility function never occurs for $Q_0 \leq I_0$. For simplicity, below we assume that $\bar{Q}_0 = +\infty$. Note that,

1) When $Q_0 \leq I_0$, $u_0'(Q_0) = 1 + \mu_2 e^{(I_0 - Q_0)} > 0$. Thus, $u_0(I_0) > u_0(Q_0)$, $\forall 0 \leq Q_0 < I_0$.
2) When $Q_0 \geq I_0$, $u_0'(Q_0) = 1 - 2\mu_1(Q_0 - I_0)$. Note that u_0 is continuous in $[I_0, \bar{Q}_0]$. Then, for $I_0 \leq Q_0 < \frac{1}{2\mu_1} + I_0$, u_0 is an increasing function.

Furthermore, when $Q_0 > \frac{1}{2\mu_1} + I_0$, u_0 is a decreasing function. Hence, u_0 achieves its maximum value at $Q_0 = \frac{1}{2\mu_1} + I_0$. Thus, $r_0^*(\mathbf{p}_{-0}) = \frac{1}{2\mu_1} + I_0$ is the best-response correspondence of the primary user utility function. Since $I_0 = \sum_{j=1}^K h_{pj}^2 \rho_{sp}^2 p_j$, we have

1) *Positivity:* $r_0^*(\mathbf{p}_{-0}) > 0$, $\forall \mathbf{p} \in P$.
2) *Monotonicity:* Given $\mathbf{p} \geq \mathbf{p}'$, $r_0^*(\mathbf{p}_{-0}) \geq r_0^*(\mathbf{p}'_{-0})$.
3) *Scalability:* Given $\forall\lambda > 1$, $\lambda r_0^*(\mathbf{p}_{-0}) = \lambda\frac{1}{2\mu_1} + \lambda I_0$ and $r_0^*(\lambda\mathbf{p}_{-0}) = \frac{1}{2\mu_1} + \lambda I_0$. Thus, $\lambda r_0^*(\mathbf{p}_{-0}) > r_0^*(\lambda\mathbf{p}_{-0})$, for $\lambda > 1$.

Therefore, the best-response correspondence of the primary user is a standard function. In practice, since $\bar{Q}_0$ is finite, when $\frac{1}{2\mu_1} + I_0 \geq \bar{Q}_0$, the primary user sets the interference cap at $\bar{Q}_0$. However, the NE is still unique even in this case. In this situation, the primary user cannot afford this amount of secondary user interference even when they are not operating at the energy efficient mode. Hence, the total interference from the secondary users exceeds the maximum amount that the primary user can tolerate. It is to be noted that, such an operating point is undesirable from our system point-of-view in which the primary users need to meet their required QoS level first and foremost. Although, we do not delve into possible resolutions to this problem in the current paper, a simple solution can be suggested in which the primary system uses a special beacon signal to indicate when secondary system needs to absolutely back-off its transmit powers.

B. The Power Control Game with the LMMSE Secondary Receiver

In this generalization, we assume that the secondary-user system is equipped with an LMMSE receiver, while that of primary-user system is an MF receiver[2]. The signal received at the secondary-system receiver can be written as

$$r(t) = \sum_{k=1}^K A_k b_k s_k(t) + \Theta A_0 b_0 s_0(t) + \sigma n(t),$$

where $A_k = h_{sk}\sqrt{p_k}$, b_k and $s_k(t)$ are the k-th secondary user's received signal amplitude, transmitted symbol and signalling waveform, respectively. Further, $A_0 = h_{s0}\sqrt{P_0}$, b_0 and $s_0(t)$ are the primary user's received signal amplitude, transmitted symbol and the signalling waveform, respectively, and $n(t)$ is white Gaussian noise with unit variance. The random variable Θ is Bernoulli with a parameter p and is introduced to denote that in an overlay system the primary user interferes with secondary transmissions only when secondary users make an error in detecting white spaces. Note that, in an overlay cognitive radio system, secondary users seek white spaces to transmit via spectrum sharing. However, there may be sensing errors that can lead to erroneous detection of white spaces with a probability p. In other words, p is the probability of collision of the transmissions from a secondary user with that of the primary user. On the other hand, for an underlay system, we may assume that $\Theta = 1$ with probability 1 since secondary users are assumed to be always active in the spectrum simultaneously with the primary user. By projecting $r(t)$ onto a set of N orthonormal signals $\{\psi_1, \psi_2, ..., \psi_N\}$

[2]Since we have assumed only one primary user, the LMMSE and the MF is the same at the primary receiver.

defined on $[0, T]$, where T is the symbol duration, we obtain the following discrete time model:

$$\mathbf{r} = \sum_{j=1}^{K} A_j b_j \mathbf{s_j} + \Theta A_0 b_0 \mathbf{s_0} + \sigma \mathbf{m},$$

where $\mathbf{s}_k = [s_{k1}, ..., s_{kn}]$ with $s_{kl} = \int_0^T s_k(t)\psi_l(t)dt, \ \forall k = 0, 1, 2..., K$ and $\mathbf{m}$ is an N-dimensional Gaussian vector with independent, zero-mean and unit-variance components.

For detecting the k-th secondary user, the common secondary receiver employs the following LMMSE filter:

$$\min_{\mathbf{w}_k, \mathbf{s}_0} E[(b_k - \mathbf{w}_k^T \mathbf{r})^2] \quad s.t. \ E[\Theta]\mathbf{S}^T \mathbf{s}_0 = \underline{\rho}_p, \tag{6}$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_K]$ is an $N \times K$ matrix and $\underline{\rho}_p = [\rho_{p1}, \rho_{p2}, ..., \rho_{pK}]^T$ is the effective cross-correlation vector between the primary user and secondary users. Note that, $E[\mathbf{r}\mathbf{r}^T] = \sum_{j=1}^K A_j^2 \mathbf{s}_j \mathbf{s}_j^T + E[\Theta^2] A_0^2 \mathbf{s_0} \mathbf{s}_0^T + \sigma^2 \mathbf{I}$, and $E[b_k \mathbf{r}] = A_k \mathbf{s}_k$.

We assume that all secondary users are in the same system, so that cross-correlations ρ_{jk}, for $j, k \in \{1, \cdots, K\}$, among them are known to all secondary users. It is also assumed that secondary system may be able to estimate the cross-correlation ρ_{pk} between the primary user and the k-th secondary user.

The LMMSE filter solution is given by

$$\mathbf{w_k} = E[\mathbf{r}\mathbf{r}^T]^{-1} E[b_k \mathbf{r}] = \frac{A_k}{1 + A_k^2 \mathbf{s}_k^T \mathbf{\Sigma}_k^{-1} \mathbf{s}_k} \mathbf{\Sigma}_k^{-1} \mathbf{s}_k,$$

where

$$\begin{aligned} \mathbf{\Sigma}_k &= \sigma^2 \mathbf{I} + E[\Theta^2] A_0^2 \left((E[\Theta])^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p \right) \left((E[\Theta])^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p \right)^T \\ &\quad + \sum_{j=1, j \neq k}^{K} A_j^2 \mathbf{s}_j \mathbf{s}_j^T \\ &= \sigma^2 \mathbf{I} + p A_0^2 \left(p^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p \right) \left(p^{-1} (\mathbf{S}^T)^+ \underline{\rho}_p \right)^T \\ &\quad + \sum_{j=1, j \neq k}^{K} A_j^2 \mathbf{s}_j \mathbf{s}_j^T \end{aligned} \tag{7}$$

where $\mathbf{S}^+$ is the pseudo-inverse of $\mathbf{S}$. Finally, the k-th secondary user's SINR at the secondary receiver can be written as

$$\gamma_k^{(s)} = A_k^2 \mathbf{s_k}^T \mathbf{\Sigma}_k^{-1} \mathbf{s_k} = h_{sk}^2 p_k \mathbf{s_k}^T \mathbf{\Sigma}_k^{-1} \mathbf{s_k}, \tag{8}$$

where $\mathbf{\Sigma}_k$ is give by (7). Note that, as was the case with the MF-based secondary-system receiver, $\frac{\partial \gamma_k^{(s)}}{\partial p_k} = \frac{\gamma_k^{(s)}}{p_k}$ even in this case, since $\mathbf{s}_k$, $\mathbf{\Sigma_k}^{-1}$ and h_{sk} are independent of p_k.

It should be pointed out that the only difference between the above power control game with the LMMSE receiver and that with the MF receiver in Section III-A is in the received SINR expression for secondary users. It is well known that the linear MMSE receiver maximizes the output SINR [21]. Thus, we may expect that under the same target SINR constraints in the primary system, the linear MMSE receiver may lead to secondary radios to transmit at a lower power level than that with the MF receiver.

1) Existence of a Nash Equilibrium with the LMMSE Receiver: As discussed in Section III-A1, a Nash equilibrium exists in game $G = (K, \mathcal{P}, u_k(.))$, if the action set $\mathcal{P}_k$ of k-th user, for all $k = 0, 1, \cdots, K$, is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^N$, and $u_k(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasi-concave in p_k. Again, we remind that $\mathcal{P}_0 = \mathcal{Q}$ and $p_0 = Q_0$ for the primary user.

Since the only difference here, compared to the discussion in Section III-A1, is in the LMMSE-based secondary receiver (as opposed to the MF-based secondary-system receiver), the only condition that we need to establish anew is the quasi-concavity of secondary-user utility (3) as a function of its power action p_k, when the receiver is based on an LMMSE detector. However, this quasi-concavity of the utility (3) with the LMMSE receiver has been established in [12] for a traditional wireless network. The only difference here is that of the interference term due to the primary user. This extra interference term from primary user in our proposed game, however, does not alter the quasi-concavity of the secondary user utility function since it is treated as an additional noise term by the secondary-system receiver.

Note that, since the secondary-system receiver does not influence the behavior of the primary user utility function, the quasi-concavity of the primary user utility function shown in Section III-A1 still holds with the LMMSE secondary receiver. It then follows that there exists at least one NE in the above power control game with the LMMSE-based secondary-system receiver.

2) Uniqueness of the NE with the LMMSE Receiver: To establish the uniqueness of the NE of the proposed cognitive power-control game with the LMMSE-based secondary receiver, first we show that the best-response correspondence $\mathbf{r}(\mathbf{p}) = (r_0(\mathbf{p}), r_1(\mathbf{p}), ..., r_K(\mathbf{p}))$ is a standard function, where $r_0(\mathbf{p})$ represents the primary user's best-response correspondence and $r_k(\mathbf{p})$, for $k = 1, \cdots, K$ represents the $k-$th secondary user's best-response correspondence.

Since the primary user's utility function stays the same as that in Section III-A1, the best-response correspondence of the primary user is still a standard function as was shown in Section III-A2. Hence, we only need to show that the best-response correspondence of the secondary users is a standard function. From the discussion in Section III-A2, the best-response correspondence of the k-th secondary user is the transmit power which provides it with the optimum SINR γ^*, where γ^* is the solution to $f'(\gamma^*)\gamma^* = f(\gamma^*)$ with $f(.)$ being the efficiency function defined earlier. Hence, from Section III-A2, the best-response correspondence of the k-th secondary user is,

$$r_k^*(\mathbf{p}) \quad = \quad \frac{\gamma^* I_k}{h_{sk}^2}, \tag{9}$$

where $I_k = \left(\mathbf{s}_k^T \mathbf{\Sigma}_k^{-1} \mathbf{s}_k \right)^{-1}$.

In Appendix A we show that indeed the best-response correspondence (9) is a standard function when the secondary-system receiver is based on an LMMSE detector. Hence, it follows that the Nash equilibrium in the power-control game with the LMMSE-based secondary receiver is unique. Again, it is worth mentioning that in the discussion in Appendix A we have assumed that $\bar{P}_k = +\infty$ and $\bar{Q}_0 = +\infty$. When

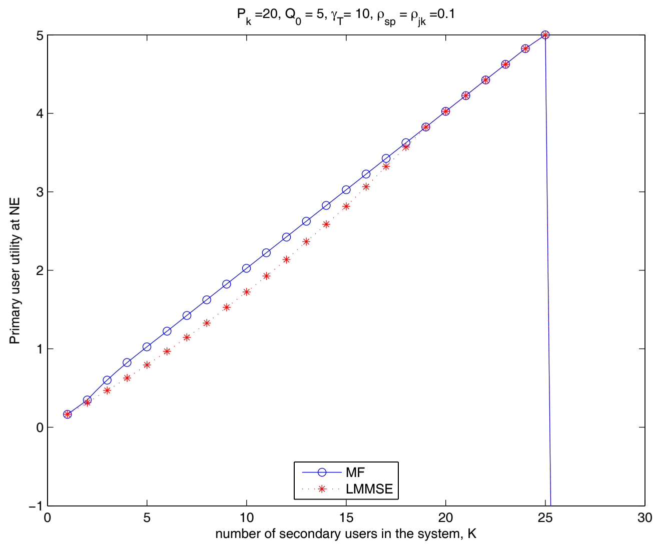

Fig. 3. Primary user's utility at the NE. $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $\bar{\gamma}_0 = 10$, $\rho_{sp} = \rho_{ps} = 0.1$ and $\rho_{jk} = 0.1$ for all $j, k \in \{1, \cdots, K\}$.

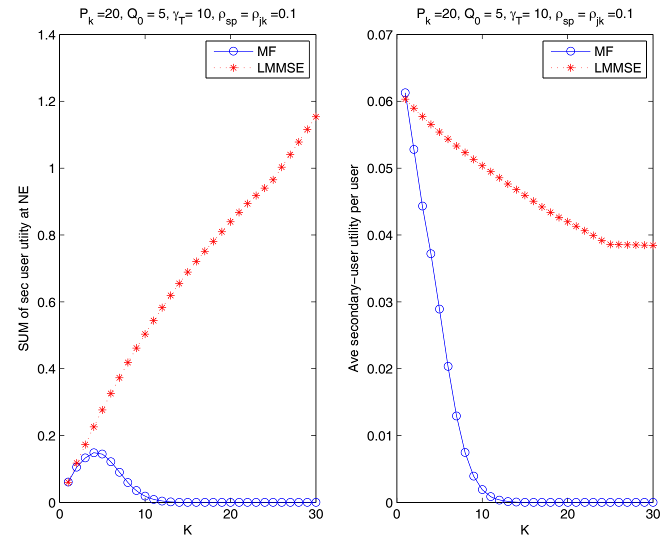

Fig. 4. Sum and average secondary user utilities at the NE. $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $\bar{\gamma}_0 = 10$, $\rho_{sp} = \rho_{ps} = 0.1$ and $\rho_{jk} = 0.1$ for all $j, k \in \{1, \cdots, K\}$.

$\bar{P}_k$ and $\bar{Q}_0$ are finite, the best-response correspondence of the k-th secondary user is given by $\min\left(\bar{P}_k, p_k^*\right)$ where p_k^* is the transmit power which provides the k-th secondary user with the optimum SINR γ^*. Similarly, the best-response correspondence of the primary user is $\min\left(\bar{Q}_0, Q_0^*\right)$ where $Q_0^* = \frac{1}{2\mu_1} + I_0$. In this case, it can be shown that the NE is still unique.

IV. Performance Results and Discussion

In this section, we investigate the behavior of a dynamic spectrum leasing cognitive radio network based on our proposed game-theoretical framework via simulations. Our objective is to delineate the key characteristics and trends emerging from our framework for spectrum leasing. Following parameter values are used in all numerical simulations unless stated otherwise: $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $h_{pk} = 1$, $,\forall k \in K$, $h_{sk} = 1$, $,\forall k \in K$, $h_{s0} = 1$, $h_{p0} = 1$, $\rho_{sp} = \rho_{ps} = 0.1$, $M = 80$, $\bar{\gamma}_0 = 10$, $\mu_1 = 10$, $\mu_2 = 100$, and $\sigma^2 = 1$. In Figs. 3 and 4 below, we first describe the behavior of the proposed system with $\rho_{jk} = 0.1$, for all $j, k \in \{1, \cdots, K\}$ among secondary users.

Figure 3 shows the primary user utility at the NE, as a function of the number of users K in the secondary system. According to Fig. 3, with the MF-based receiver only for $K \leq 3$, all secondary users are able to achieve $SINR = \gamma^*$ that maximizes their utility. When $K > 3$, the network cannot support these secondary users, and as a result, no secondary user can achieve the optimal SINR γ^*. Thus, all secondary users are forced to transmit at their maximum possible power level of $\bar{P}_k$. It can be shown that the primary user's utility at its best-response is $u_0 = \frac{1}{4\mu_1} + I_0$. Since $p_k = \bar{P}_k$ when $K > 3$, the total secondary interference seen at the primary receiver is $I_0 = K\bar{P}_k h_{pk}^2 \rho_{sp}^2$. Hence, I_0 increases linearly with K after this point and as a result the primary user's utility at the NE also increases as a linear function in K. However, when $K > 25$, we have $\bar{Q}_0 < I_0$ and the primary user's utility is severely penalized by the exponential pricing function. Thus, from the point of view of the primary system $K > 25$ would be a region of operation that is extremely undesirable. Hence, the secondary system should not operate in this region (i.e. use $K \geq 26$). On the other hand, with the LMMSE-based receiver, Fig. 3 shows that all secondary users can achieve optimal SINR γ^* at the NE until $K \leq 18$. This is only one of the, and expected, advantages of the LMMSE-based secondary receiver over that based on the MF. However, as seen from Fig. 3 for $K \leq 18$ the primary user utility at the NE with the LMMSE receiver is also less than that with the MF-based receiver. Recall that, as we mentioned earlier, the primary user utility can be interpreted as proportional to the payments the secondary system need to provide for using its spectrum. This shows that if secondary system can better manage its transmit powers, and thus reduce the total interference I_0 it causes to the primary user, by employing a more powerful detector (in this case the LMMSE), that may lead to reduced payments.

Figure 4 shows the total sum-utility as well as per-user average utility achieved by the secondary system at the Nash equilibrium as a function of number of secondary users K in the system. As seen from Fig. 4, the sum-utility of all secondary users with MF-based receiver has a unique maximum at $K = 4$. As the secondary system attempts to include more than $K = 4$ users into the same spectrum band, the sum-utility of the secondary system starts to monotonically decrease. This is because, as the number of secondary users increases, each secondary user, as well as the primary user, sees more interference due to the additional secondary users. Hence to achieve the same optimum SINR, each secondary user has to transmit at a higher power than that with smaller number of secondary users in the system. As can be seen from the right hand side of Fig. 4, this then causes the average utility per secondary user achieved by the secondary system to decrease. This monotonic reduction in per-user utility with K is true for both types of receivers considered. Note that, however, with the LMMSE-based receiver although the average utility per secondary user monotonically decays with increasing K, this

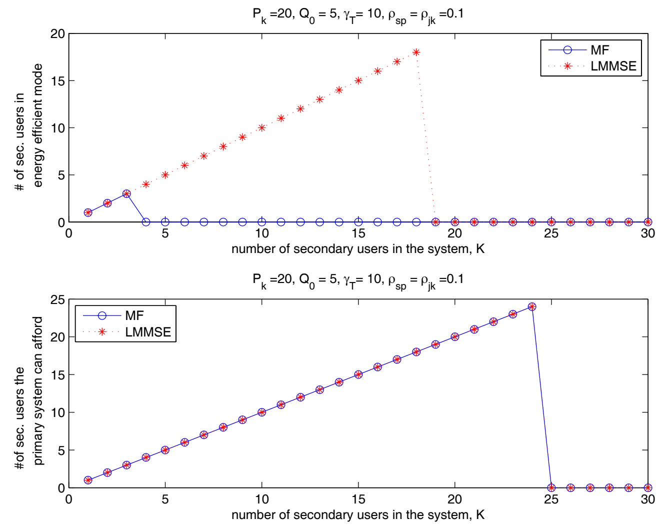

Fig. 5. Number of secondary users in energy efficient transmission mode and the number of affordable users by the primary system. $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $\bar{\gamma}_0 = 10$, $\rho_{sp} = \rho_{ps} = 0.1$ and $\rho_{jk} = 0.1$ for all $j, k \in \{1, \cdots, K\}$.

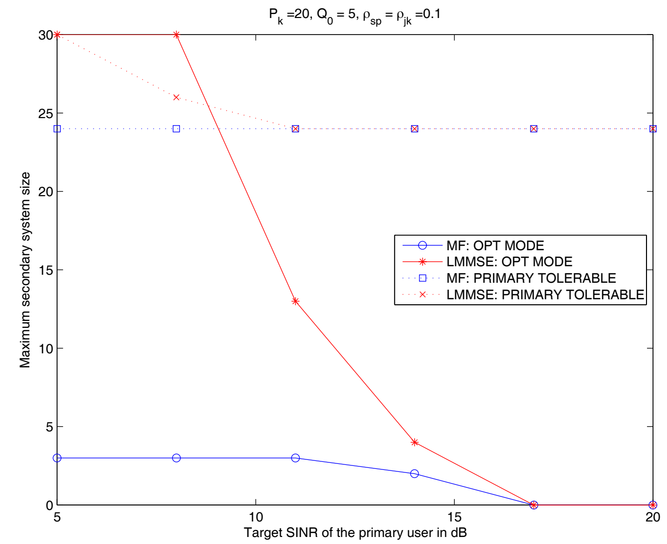

Fig. 6. Influence of the primary user's target SINR $\bar{\gamma}_0$ on the tolerable and energy-efficient secondary system size. $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $\rho_{sp} = \rho_{ps} = 0.1$ and $\rho_{jk} = 0.1$ for all $j, k \in \{1, \cdots, K\}$.

is more than offset by the increased number of users in the system. As a result the sum-utility monotonically increases in the case of the LMMSE-based receiver. However, with the MF-based receiver this is only true for as long as $K < 4$. For $K \geq 5$ the per-user average utility has suffered too much and as a result the sum-utility also decreases. Observe also that, with the LMMSE-based receiver, the average utility per secondary user is always better than that with the MF-based receiver.

Figure 5 shows the number of secondary users that can achieve the optimum SINR γ^* at the Nash equilibrium of the combined system and the number of affordable secondary users by the primary system, as a function of the total number of secondary users K. It is well known that the LMMSE receiver has a better multiuser interference suppression capability than the MF receiver. Thus, in the same interference environment, secondary users are expected to achieve the optimal SINR γ^* with lower transmit power levels when the LMMSE receiver is employed. In return, the primary user will cause less interference to secondary users. This is because in order to achieve its transmission quality the primary user needs to increase its transmission power as the secondary users' interference increases. When a secondary user can transmit at a power level that achieves a received SINR of γ^*, we call that user to be in the energy-efficient transmission mode. Thus, the LMMSE receiver's superior interference suppression capability lead to a system in which more secondary users can operate in the energy-efficient mode. This is shown on the top half of Fig. 5. It should be noted that in Fig. 5 either all users in the system are in the same energy-efficient mode or none-of-them are. This is due to the fact that we have assumed AWGN channels and identical parameters for all secondary users. As is seen from the top half of Fig. 5, with the MF receiver only a system with up to $K = 3$ secondary users can achieve energy efficient transmissions. On the other hand, as predicted above, with the LMMSE-based receiver up to $K = 18$ secondary users can achieve the optimum SINR γ^* at the NE.

The bottom half of Fig. 5 shows the maximum number of secondary users that can be afforded by the primary user system so that $\bar{Q}_0 \geq I_0$. As can be seen from Fig. 5, for the assumed parameter values this maximum number of affordable secondary users by the primary system is the same for both types of secondary-system receivers. This is because, for this set of parameter values, both systems reach the condition $\bar{Q}_0 < I_0$ in the region of operation where none of the secondary users achieve optimal SINR $\bar{\gamma}^*$. As a result, all secondary users are transmitting at the maximum allowed transmit power of $p_k = \bar{P}_k$ making I_0 directly proportional to K. However, due to the superior interference suppression capability of the LMMSE receiver, one may expect for other combinations of parameter values LMMSE-based secondary receiver will be able to tolerate more secondary users than that with the MF receiver. Indeed, this is true as we will see below.

In the proposed system model , we have assumed that the primary user has a target SINR, denoted by $\bar{\gamma}_0$, that is determined by its transmission quality requirement. If total interference I_0 from all secondary users is below the interference cap Q_0 the primary user sets, the primary user can achieve this target SINR and still gain a positive utility. Otherwise, the primary user cannot achieve its transmission quality and its utility decays fast (in fact, exponentially). The target SINR in conjunction with instantaneous interference cap Q_0 determines the primary user's transmission power p_0 (see (1)). Higher the transmission power p_0, higher the interference to the secondary users it creates (for a fixed ρ_{sp}). Thus, the target SINR $\bar{\gamma}_0$ determines the flexibility the primary user has in terms of sharing its spectrum with the secondary users: lower the $\bar{\gamma}_0$ value, more secondary users can be tolerable in the system and vice versa.

Figure 6 shows the dependance of affordable secondary system size on the target SINR of the primary system. We have shown both maximum secondary system size so that

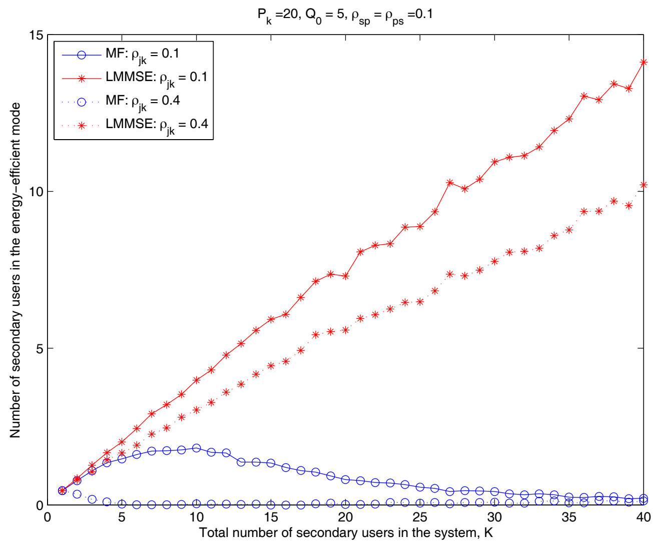

Fig. 7. Number of secondary users in the energy-efficient transmission mode when all channels are standard Rayleigh fading. $\bar{P}_k = 20$, $\bar{\gamma}_0 = 10$dB, $\bar{Q}_0 = 5$, $\rho_{sp} = \rho_{ps} = 0.1$.

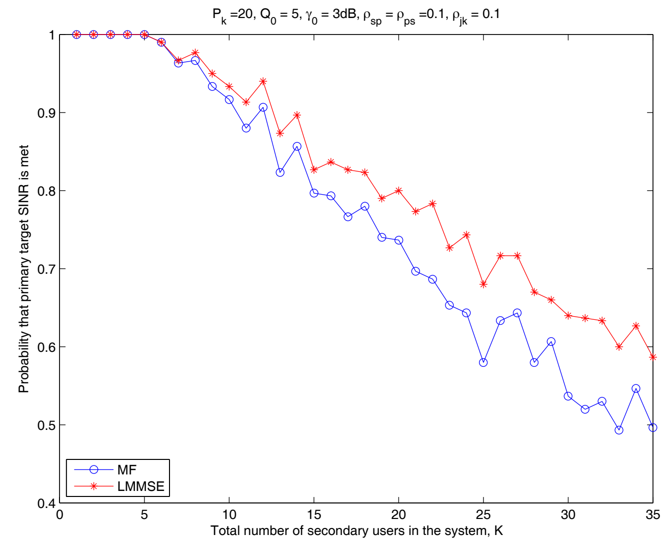

Fig. 8. Probability that the target SINR requirement of the primary user is met when all channels are standard Rayleigh fading. $\bar{P}_k = 20$, $\bar{Q}_0 = 5$, $\bar{\gamma}_0 = 3$dB, $\rho_{sp} = \rho_{ps} = 0.1$ and $\rho_{jk} = 0.1$ for all $j, k \in \{1, \cdots, K\}$.

all users can transmit in energy-efficient mode as well as the maximum tolerable secondary system size before the primary user transmission quality is compromised. Figure 6 confirms the expected behavior: As $\bar{\gamma}_0$ increases the both maximum tolerable system size as well as the maximum energy-efficient system size decreases. Moreover, in both cases, the LMMSE receiver allows significantly larger secondary systems to be supported as compared to that with the MF-based receiver unless the target SINR requirement is too high. Again, this is due to the better multiuser interference suppression capability of the LMMSE receiver that allows secondary users to achieve γ^* with lower transmit power, causing reduced interference at the primary receiver. In return, the primary user will cause less interference to secondary users since it is able to achieve its required quality-of-service at a reduced transmit power level. This leads to a system in which more secondary users can achieve energy efficient transmissions. In particular, as can be seen from Fig. 6, when $\bar{\gamma}_0$ is low, the LMMSE-based system allows all secondary users to operate in energy-efficient mode till the maximum primary-tolerable limit. On the other hand with the MF-based receiver, the number of users that can operate in energy-efficient mode falls far below the limit at which the primary's quality of service is compromised. Overall, Fig. 6 shows that the system with the LMMSE receiver can support more secondary users to achieve the energy efficient transmissions. Further, the LMMSE receiver allows more secondary users to be admitted to the system with some level of transmit power, although not necessarily at the energy efficient level, before primary system cannot afford them.

In all examples above, we saw that either all or none of the secondary users in a system achieve the energy-efficient optimum SINR γ^*. This was expected because we assumed all channels to be AWGN so that all users in the system are received at the same power level. In Fig. 7 we have shown the number of secondary users operating in energy-efficient mode when all are assumed to be fading channels. In particular, all channel coefficients are assumed to be standard Rayleigh distributed (unit second moment), and the results are averaged over 100 independent channel realizations. Note that, the cross-correlations among all secondary users are assumed to be the same in Fig. 7, and equal to either $\rho_{jk} = 0.1$ or $\rho_{jk} = 0.4$, for $j, k \in \{1, \cdots, K\}$. All other parameters used in Fig. 7 are the same as that in Fig. 5. Figure 7 shows how fading can effect the number of energy-optimum users in the system. The main observation is that the average number of users in energy-efficient mode is not hard-limited, as in the case of AWGN channels. For example, with the LMMSE receiver, as long as $K \leq 18$ all the secondary users in the system were able to transmit at the energy-efficient mode in an AWGN channel, but for $K > 18$ none of the users were able to achieve energy-efficient mode (see Fig. 5). However, when there is channel fading, even for $K = 30$, on average about 10 users can transmit in energy-efficient mode.

The same is true for the maximum tolerable system size. Depending on the fading realizations, the primary user may or may not be able to tolerate a particular number of secondary users in the system. Fig. 8 shows the probability that the primary user can tolerate (i.e. $I_0 \leq Q_0$ at the NE) a particular size secondary user system. In Fig. 8 we have assumed that $\bar{\gamma}_0 = 3$dB and all signalling correlations are 0.1. Each point in the figure is obtained by averaging over 300 independent fading realizations for all channels. As expected when the total number of secondary users in the system increases, the probability the primary SINR target is met decreases. However, for any secondary system size there is always a possibility that for some fading values the primary system may be able to tolerate that many secondary users. Moreover, Fig. 8 shows that the LMMSE based secondary receiver ensures a higher probability of primary system being satisfied with its quality as against that with the MF-based secondary receiver.

V. Conclusion

In this paper, we proposed the novel concept of dynamic spectrum leasing as an alternative to hierarchical DSA to improve spectrum utilization efficiency. The proposal is to be viewed as a technique to be used in light of the dynamic exclusive-use spectrum rights model identified by the FCC. In the proposed dynamic spectrum leasing framework, the primary users who own spectrum property rights have an incentive to allow secondary users to operate in their spectrum bands whenever possible to the maximum extent because their compensation is to be proportional to that. This is in contrast to the traditional hierarchical DSA model that is being considered by many in the existing literature. In his work, we have also developed a game theoretical framework to facilitate dynamic spectrum leasing in a cognitive radio network. The main difference of this game model, compared to game models used for hierarchical DSA, is that here primary users are also included as active decision makers in the same non-cooperative power control game. The primary users are to be rewarded for allowing the secondary users to access their spectrum. Thus, we proposed a new primary utility function that is proportional to the amount of interference that the primary user is willing to tolerate from all secondary users, while secondary user utility was their throughput per unit power. Thus, primary user's strategy in this game is to choose the best interference cap at any given time, while that of secondary users is to adapt their transmit powers. We established that this primary-secondary user power control game has a unique Nash equilibrium, thereby allowing a round-robin power adaptation to converge to the NE of the game. This was shown to be true with either an MF-based or LMMSE-based secondary system receiver. Through a series of simulated examples, we showed that how the proposed game formulation can provide useful design guidelines for dynamic spectrum leasing. In particular, we showed that with LMMSE receiver one may expect more secondary users to achieve energy-efficient transmissions for the same maximum interference cap of the primary user. Moreover, the probability that a given secondary system size will be tolerable by the primary system was also improved significantly by using an LMMSE receiver. This may greatly facilitate the co-existence of the two systems.

We believe dynamic spectrum leasing to be more attractive than hierarchical DSA in the long run since in hierarchical DSA there is no incentive for the spectrum license holders to care about secondary transmissions in any way. Moreover, current hierarchical DSA can be considered as a degenerate special case of the proposed DSL framework, thus making it the more general approach. Future work in this topic will consider the generalization of our proposed framework to more realistic cognitive radio environments consisting of multiple primary user systems as well as infrastructure-less (ad-hoc) networks.

Appendix A
Best-Response of the k-th Secondary User with the LMMSE Receiver

In showing that the best-response function of the k-th secondary user is a standard function when the secondary receiver is based on the LMMSE detector, we will need the following result that we proved in [20], and quoted here for completeness.

Proposition If two $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ are both real, symmetric and positive definite, such that $\mathbf{B} - \mathbf{A} \geq \mathbf{0}$ (i.e. $\mathbf{B} - \mathbf{A}$ is positive semi-definite), then $\mathbf{A}^{-1} - \mathbf{B}^{-1} \geq \mathbf{0}$. In particular, when $\mathbf{B} - \mathbf{A} > \mathbf{0}$, than $\mathbf{A}^{-1} - \mathbf{B}^{-1} > \mathbf{0}$.

In the following we show that the best-response $r_k^*(\mathbf{p}) = \frac{\gamma^* I_k}{h_{sk}^2}$ of the k-th secondary user, for $k = 1, \cdots, K$, given in (9) satisfies the three sufficient conditions for it to be a standard function.

1) positivity: Since $\gamma^* > 0, I_k > 0$, the best response correspondence of the k-th secondary user satisfies $r_k^*(\mathbf{p}) = \frac{\gamma^* I_k}{h_{sk}^2} > 0$.
2) monotonicity: By following a proof similar to that in [20], if $\mathbf{p} \geq \mathbf{p}'$, then $p_k > p_{k'}$, for $\forall k = 0, 1, ..., K$. Hence $\mathbf{\Sigma_k}(\mathbf{p}) - \mathbf{\Sigma_k}(\mathbf{p}') \geq 0$. From the above Proposition, we then have

$$\mathbf{\Sigma_k}(\mathbf{p})^{-1} - \mathbf{\Sigma_k}(\mathbf{p}')^{-1} \leq 0 \Leftrightarrow \mathbf{s}_k^T \mathbf{\Sigma_k}(\mathbf{p})^{-1} \mathbf{s}_k - \mathbf{s}_k^T \mathbf{\Sigma_k}(\mathbf{p}')^{-1} \mathbf{s}_k \leq 0,$$

which implies that $I_k(\mathbf{p}') \leq I_k(\mathbf{p})$. Thus,

$$r_k^*(\mathbf{p}) = \frac{\gamma^* I_k(\mathbf{p})}{h_{sk}^2} \geq \frac{\gamma^* I_k(\mathbf{p}')}{h_{sk}^2} = r_k^*(\mathbf{p}').$$

3) scalability: For $\mu > 1$,

$$\mu r_k^*(\mathbf{p}) = \frac{\mu \gamma^* I_k(\mathbf{p})}{h_{sk}^2}, \quad \text{and} \quad r_k^*(\mu \mathbf{p}) = \frac{\gamma^* I_k(\mu \mathbf{p})}{h_{sk}^2}.$$

From [20], by using the above Proposition, we have that $\mu I_k(\mathbf{p}) > I_k(\mu \mathbf{p})$. Hence $\mu r_k^*(\mathbf{p}) > r_k^*(\mu \mathbf{p})$.

Hence, the best-response of the secondary users is a standard function with the LMMSE-based receiver.

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