Optimal and Low-complexity Algorithms for Dynamic Spectrum Access in Centralized Cognitive Radio Networks with Fading Channels

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***Abstract*—In this paper, we develop a centralized spectrum sensing and Dynamic Spectrum Access (DSA) scheme for secondary users (SUs) in a Cognitive Radio (CR) network. Assuming that the primary channel occupancy follows a Markovian evolution, the channel sensing problem is modeled as a Partially Observable Markov Decision Process (POMDP). We assume that each SU can sense only one channel at a time by using energy detection, and the sensing outcomes are then reported to a central unit, called the secondary system decision center (SSDC), that determines the channel sensing/accessing policies. We derive both the optimal channel assignment policy for secondary users to sense the primary channels, and the optimal channel access rule. Our proposed optimal sensing and accessing policies alleviate many shortcomings and limitations of existing proposals: (a) ours allows fully utilizing all available primary spectrum white spaces, (b) our model, and thus the proposed solution, exploits the temporal and spatial diversity across different primary channels, and (c) is based on realistic local sensing decisions rather than complete knowledge of primary signalling structure. As an alternative to the high complexity of the optimal channel sensing policy, a suboptimal sensing policy is obtained by using the Hungarian algorithm iteratively, which reduces the complexity of the channel assignment from an exponential to a polynomial order. We also propose a heuristic algorithm that reduces the complexity of the sensing policy further to a linear order. The simulation results show that the proposed algorithms achieve a near-optimal performance with a significant reduction in computational time.**

***Index Terms*—Cognitive radio, dynamic spectrum access (DSA), partially observable Markov decision processes (POMDP), Hungarian algorithm.**

I. Introduction

OPPORTUNISTIC Spectrum Access (OSA) is emerging as one of the Dynamic Spectrum Access (DSA) techniques that can mitigate the underutilization of the spectrum bands. Such DSA techniques can be implemented by using Cognitive Radio (CR) devices which are supposed to be equipped with the ability to learn and adapt to their RF environment. A set of cognitive radios may form a secondary network that coexists with the primary licensed users and shares the spectrum opportunistically. This is referred to as the spectrum interweave which permits the secondary users to communicate by using the spectrum holes in the primary bands [1].

In order to achieve successful spectrum coexistence, however, the cognitive users should be able to *correctly* identify the spectrum holes and to transmit without interfering with the primary users (PUs). Of course, this may not be always possible: The secondary users can make wrong decisions about the occupancy of the spectrum holes due to receiver noise and fading in the wireless channels. Sophisticated detectors, such as the matched filter and the cyclostationary detector, may be employed by cognitive users for obtaining a better estimate of the primary channels' status, as we describe in the accompanying paper. However, this would require some information about the primary signal leading to additional complexity at the cognitive devices. On the other hand, SUs are usually intended to operate in different RF environments, therefore, they are aimed to detect any existing primary signal, irrespective of its characteristics. In this case, the cognitive users do not assume any knowledge about the primary signal and they may employ energy detection as an optimal technique to perform the spectrum sensing, as we describe next throughout this paper.

A secondary user can obtain a better estimate about the primary channels occupancy by basing its decisions not only on the current but also on the the past observations of the channels, if the primary traffic exhibits some temporal correlation. In particular, if a channel is characterized at each instant to be either idle (state 0) or busy (state 1), the state transitions may be modeled as a Markov chain, and the optimal sensing policy can be obtained by modeling the system as a Partially Observable Marov Decision Process (POMDP). This method has been studied in the past [2], [3], but the optimal solution to the POMDP is shown to be *computationally prohibitive* because of the continuum of the state space. In this case, it is more convenient to maximize a reward function at each time instant, instead of maximizing the total discounted return, thus obtaining a *myopic policy* for the POMDP problem.

In this paper, unlike [2], we assume a *centralized* CR network with a Secondary System Decision Center (SSDC) that receives, at every instant, the sensing outcomes of all the SUs and determines the sensing and accessing policies of

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SUs accordingly. The observations of the SUs are assumed to be affected by independent channel fading coefficients, which makes our proposed model more realistic than the model in [3] since we take into account the spatial and temporal variations of the wireless channels. Also, we do not restrict all the SUs to sense the same primary channel at a time, as assumed in [3]. This allows efficient exploitation of all spectrum vacancies, thus maximizing the network throughput. We design the optimal detectors of the SUs and we derive a myopic channel sensing policy that maximizes the secondary network throughput at each time instant. However, the optimal solution of the myopic sensing policy is found to be computationally expensive since it has an exponential complexity. Therefore, we apply the Hungarian algorithm iteratively in order to find a near-optimal sensing policy in a polynomial time. The iterative Hungarian extends the Hungarian algorithm [4] by allowing more than one vertex to be connected to a single vertex of the other bipartite set, which is equivalent to assigning more than one SU to sense a single primary channel when the number of SUs is larger than the number of channels. We also propose a heuristic algorithm that solves the channel assignment problem at a linear complexity order. The simulation results show that these low-complexity proposed algorithms can achieve a near-optimal policy, yet with a significant reduction in the computation time.

The remaining of this paper is organized as follows: Section II defines the system model where we describe the local sensing decisions. Sections III determines the accessing decisions at the SSDC, and Section IV presents the two algorithms for deriving the sensing policy. The simulation results are shown in Section V and we conclude this paper in Section VI.

II. System model

We assume a group of N SUs, and a collection of M primary channels. The primary channels' states are modeled as statistically identical and independent two-state (*busy* and *idle*) Markov chains. The state *busy* refers to the channel being occupied by a PU, whereas the *idle* state refers to a spectrum vacancy which can be used by SUs. We denote the true state of primary channel $m \in \{1, \cdots, M\}$ in time slot k by $S_m(k) \in \{0,1\}$. The stationary transition probability of channel m from state i to state j is defined as $p_{ij} = \Pr\{S_m(k+1) = j \mid S_m(k) = i\}, \forall i, j \in \{0,1\}$. The *transition probability matrix* of the Markov chain is denoted by $\mathbb{P} = [p_{00} \quad p_{01}; p_{10} \quad p_{11}]$.

When a SU successfully accesses a primary channel that is idle during a given time slot, the SU is assumed to receive a reward proportional to the bandwidth of that channel. If a SU accesses a primary channel in state *busy*, it will cause a collision with PUs' transmission and it gets a 0 reward in this case. The accumulated total reward of all SUs is used as a measure of the secondary system throughput over the primary channels.

In order to detect the spectrum white spaces, SUs perform spectrum sensing. We assume that the secondary CRs are equipped with only a single antenna that switches between sensing and actual communication. As a result, when a SU is performing channel sensing, it stops its data communications. It is also assumed that a single SU can only sense one primary channel at a time. As shown in Fig. 1, SUs sense primary channels during the designated sensing periods at the beginning of each time slot. It is assumed that if a PU intends to use its channel during a transmitting period, it will start to transmit from the beginning of that time slot. On the other hand, we assume that multiple SUs can simultaneously sense the same primary channel.

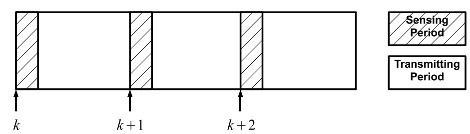

Fig. 1. Slotted time horizon with Sensing Periods and Transmitting Periods.

At each time instant k, the SSDC predicts the channel fading coefficients in the next slot $k+1$. Based on these coefficients and on the belief of the channels' state in the next time slot, the SSDC computes the sensing decisions for time $k+1$. Then, each SU senses its assigned channel and it reports its sensing outcome to the SSDC which decides which channel to access at time $k+1$. The access is scheduled among secondary users such that it guarantees equal spectrum opportunities for all SUs.

We represent the sensing decision by the $M \times N$ matrix $\mathbf{A}_k$, where $\mathbf{A}_k(m,n) \in \{0,1\}$. The secondary user n should sense channel m at time k only if $\mathbf{A}_k(m,n) = 1$. Similarly, we define the $M \times N$ matrix $\mathbf{B}_k$ to denote the accessing decision at time k.

We use $M \times N$ matrix $\mathbf{Y}_k$ to denote the collection of observation results from all SUs on their assigned primary channels at time k with $\mathbf{Y}_k(m,n) = y_{m,n}(k)$, where $y_{m,n}(k)$ is used to denote the report from SU n to the SSDC of the state of m-th primary user at time k. The SSDC uses the entries $\mathbf{Y}_k(m,n)$, such that $\mathbf{A}_k(m,n) = 1$ in order to make the access decisions at time k. The decision making architecture is summarized in Algorithm 1.

Algorithm 1 Decision making architecture

1. At each time k, based on previous knowledge of primary channels and channel observations, the SSDC sends out the sensing decisions $\mathbf{A}_k$ to all SUs.
2. SUs perform channel sensing according to $\mathbf{A}_k$ and sensing result $\mathbf{Y}_k$ is reported back to the SSDC.
3. Based on the channel sensing result $\mathbf{Y}_k$, SSDC sends out the accessing decisions $\mathbf{B}_k$ to all SUs.
4. SUs access primary channels according to $\mathbf{B}_k$.
5. For $k \to k+1$, repeat 1 through 5.

When sensing a channel m at time k, the SU n gets the observation $r_{m,n}(k)$ defined as:

$$r_{m,n}(k) = \begin{cases} h_{m,n}(k)x_m(k) + w_n(k) & \text{if} \quad \mathcal{H}_0 : S_m(k) = 1 \\ w_n(k) & \text{if} \quad \mathcal{H}_1 : S_m(k) = 0 \end{cases}, \tag{1}$$

where $x_m(k)$ is the transmitted primary signal, $w_n(k)$ is a zero-mean Gaussian noise with variance σ_w^2, and $h_{m,n}(k)$ is the fading coefficient between the m-th primary transmitter and the n-th secondary receiver at time k. The channel coefficient $h_{m,n}(k)$ is assumed to be zero-mean Gaussian distributed with variance σ_h^2. We assume that the SSDC has perfect knowledge of all channel coefficients at each time k. Since the SU does not have knowledge about the primary signal, we model $x_m(k)$ as a zero-mean Gaussian random variable with variance σ_x^2.

Instead of transmitting the observation $r_{m,n}(k)$ to the SSDC, we assume that SUs report an estimate of the primary channel state $S_m(k)$, based on the observation $r_{m,n}(k)$. The state estimate is denoted by $y_{m,n}(k)$ and it is obtained by using a *maximum a posteriori* (MAP) detector for the observation in (1). Therefore, $S_m(k) \in \{0,1\}$ and $y_{m,n}(k) \in \{0,1\}$ can be modeled, respectively, as the input and output of a *Binary Asymmetric Channel* (BAC) having crossover probabilities $\lambda_k^0(m,n)$ and $\lambda_k^1(m,n)$ under hypotheses $\mathcal{H}_1 : \{S_m(k)=0\}$ and $\mathcal{H}_0 : \{S_m(k)=1\}$, respectively, as we illustrate in Fig. 2. We assume that transmitting $y_{m,n}(k)$'s to the SSDC is error free.

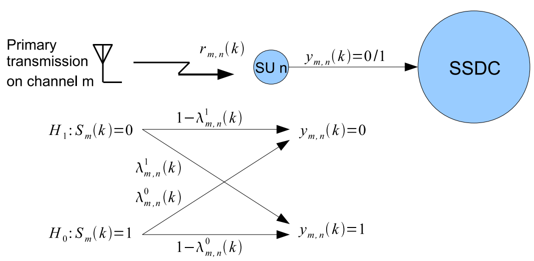

Fig. 2. SUs' reports of observations on primary channels can be modeled as Binary Asymmetric Channels.

The state estimation $y_{m,n}(k) \in \{0,1\}$ is given in (2) by using a MAP detector [5] such that $y_{m,n}(k) = \arg\max_{i\in\{0,1\}} \Pr\{S_m(k)=i|r_{m,n}(k)\}$. Then,

$$y_{m,n}(k) = \begin{cases} 0 & \text{if} \quad r_k^2(m,n) \le \eta'_{m,n}(k) \\ 1 & \text{if} \quad r_k^2(m,n) > \eta'_{m,n}(k) \end{cases}, \tag{2}$$

where

$$\eta'_{m,n}(k) = \frac{\ln\left(1+\frac{h_k^2(m,n)\sigma_x^2}{\sigma_w^2}\right) - 2\ln(\eta_m(k))}{h_k^2(m,n)\sigma_x^2\left[(h_k^2(m,n)\sigma_x^2+\sigma_w^2)\sigma_w^2\right]^{-1}}, \tag{3}$$

and $\eta_m(k) = \frac{\Pr\{S_m(k)=1\}}{\Pr\{S_m(k)=0\}}$. From (2), in this case, the MAP detector is an energy detector when $x_{m,n}(k)$ is assumed to be a Gaussian random variable.

By noting that the random variables $\frac{r^2_{m,n}(k)}{\sigma_w^2}$ and $\frac{r^2_{m,n}(k)}{\sigma_w^2+h_k^2(m,n)\sigma_x^2}$ have a χ^2-squared distribution, we may compute the crossover probabilities of the BAC sensing model as:

$$\lambda^1_{m,n}(k) = 1 - \frac{1}{\Gamma(\frac{1}{2})}\gamma\left(\frac{1}{2}, \frac{\eta'_{m,n}(k)}{2\sigma_w^2}\right), \tag{4}$$

$$\lambda^0_{m,n}(k) = \frac{1}{\Gamma(\frac{1}{2})}\gamma\left(\frac{1}{2}, \frac{\eta'_{m,n}(k)}{2(\sigma_w^2+h_k^2(m,n)\sigma_x^2)}\right), \tag{5}$$

where $\Gamma(x)$ and $\gamma(a,b)$ stand for the Gamma and the lower incomplete Gamma functions, respectively [6].

III. Centralized Access Decisions at the SSDC

In order to keep the above collision probability with PUs below a certain threshold, we apply a Neyman-Pearson type detector [5] at the SSDC to obtain the access decision rule. For simplicity, we use the variable length vector $\mathbf{y}_k(m,:) = \{y_{m,n}(k) : \forall n \in \mathcal{N}_m(k)\}$ to denote all channel sensing reports at time k, from the SUs on channel m. We define the variable length vector $\mathbf{y}_{0:k}(m,:) = \{\mathbf{y}_0(m,:),\cdots,\mathbf{y}_k(m,:)\}$ to denote the sensing results on channel m, from time 0 to k. We use vector $\mathbf{S}^m_{0:k}$ to denote the the states of channel m from time 0 to k. The set of all possible state vectors is denoted by $\mathcal{S}_c = \{0,1\}^{k+1}$.

At time k, for the m-th primary channel, the SSDC chooses one of the two possible hypotheses based on $\mathbf{y}_{0:k}(m,:)$:

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{y}_{0:k}(m,:) \sim P_{m,1} \text{ (channel } \textit{idle}\text{)} \\ \mathcal{H}_0 &: \mathbf{y}_{0:k}(m,:) \sim P_{m,0} \text{ (channel } \textit{busy}\text{)}, \end{aligned}$$

where $P_{m,1}$, and $P_{m,0}$ denote the conditional probability density of the vector $\mathbf{y}_{0:k}(m,:)$ given $S_m(k)=0$, and $S_m(k)=1$, respectively. The corresponding likelihood ratio based on $\mathbf{y}_{0:k}(m,:)$ is complicated and hard to derive in general because the length of the sequence $\mathbf{y}_{0:k}(m,:)$ increases with time. To simplify the access decision structure, we assume that the access decisions are based only on the current observations $\mathbf{y}_k(m,:)$. Then, for the m-th primary channel, the likelihood ratio is defined as:

$$\mathcal{L}(\mathbf{y}_k(m,:)) = \frac{P_{m,1}(\mathbf{y}_k(m,:))}{P_{m,0}(\mathbf{y}_k(m,:))}, \tag{6}$$

where we reuse the notation $P_{m,1}$, and $P_{m,0}$ to denote the conditional probability density of vector $\mathbf{y}_k(m,:)$ given $S_m(k)=0$, and $S_m(k)=1$, respectively.

The corresponding log-likelihood ratio is given by $\mathcal{LLR}(\mathbf{y}_k(m,:)) = \sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k) + d_m(k)$, where we define $c_{m,n}(k) = \ln\left(\frac{\lambda^1_{m,n}(k)}{1-\lambda^0_{m,n}(k)} \cdot \frac{\lambda^0_{m,n}(k)}{1-\lambda^1_{m,n}(k)}\right)$, and $d_m(k) = \sum_{n\in\mathcal{N}_m(k)} \ln\left(\frac{1-\lambda^1_{m,n}(k)}{\lambda^0_{m,n}(k)}\right)$. A sufficient statistic is $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k)$. In other words, the test $\mathcal{LLR}(\mathcal{Y}_k(m,:)) \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \tau_m(k)$ is equivalent to the test $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k) \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \tau_m(k) - d_m(k) = \tau'_m(k)$.

We use $f^0_{m,k}$, and $F^0_{m,k}$ to denote the conditional probability mass function (pmf), and the conditional cumulative distribution function (cdf) of random variable $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k)$ under hypothesis $\mathcal{H}_0$, respectively. Similarly, we use $f^j_{m,k}$, and $F^j_{m,k}$ to denote the conditional pmf, and the conditional cdf of random variable $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k)$ under hypothesis $\mathcal{H}_j$, where $j \in \{0,1\}$.

We use ζ to denote the collision probability constraint on each individual primary channel. So $\tau'_m(k)$ is chosen such that: $1-F^0_{m,k}(\tau'_m(k)) \le \zeta < 1-F^0_{m,k}(\tau'_m(k)+1)$. The randomized access decision rule is then given by

$$\tilde{\delta}_{NP}(\mathcal{Y}_k(m,:)) = \begin{cases} 1 & \text{if} \quad \sum y_{m,n}(k)c_{m,n}(k) > \tau'_m(k) \\ \gamma_m(k) & \text{if} \quad \sum y_{m,n}(k)c_{m,n}(k) = \tau'_m(k) \\ 0 & \text{if} \quad \sum y_{m,n}(k)c_{m,n}(k) < \tau'_m(k) \end{cases},$$

where the summations are with respect to $n \in \mathcal{N}_m(k)$, and $\tilde{\delta}_{NP}(\mathcal{Y}_k(m,:))$ is the probability of accessing channel m. Therefore, the SSDC decides to access channel m only if $\delta_{NP}(\mathcal{Y}_k(m,:)) = 1$, where δ_{NP} is a binomial random variable with a probability of success equal to $\tilde{\delta}_{NP}$. The randomization variable is given by $\gamma_m(k) = \frac{\zeta - F^0_{m,k}(\tau'_m(k))}{F^0_{m,k}(\tau'_m(k)-1) - F^0_{m,k}(\tau'_m(k))}$. So, the probability of detection of the white spaces is equal to:

$$P_{D,m}(k, \mathbf{A}_k) = \Pr\{\delta_{NP} = 1 \mid \mathcal{H}_1\} \tag{7}$$

$$= 1 - F^1_{m,k}(\tau'_m(k)) + \gamma_m(k) \cdot f^1_{m,k}(\tau'_m(k)). \tag{8}$$

We will use the probability of detection in the derivation of the sensing decisions at the SSDC, as we show next.

IV. Centralized Sensing Policy at the SSDC

A. Optimal Myopic Channel Sensing Policy

The objective of designing the sensing decision rule is to maximize the total secondary system reward on all channels accrued over time. To do this, we first define $b_0(m,k) = \Pr\{S_m(k) = 0 \mid \mathbf{y}_{0:k-1}(m,:)\}$, and $b_1(m,k) = 1 - b_0(m,k)$ as the belief of channel m being *idle* and *busy* at time k, given the observation history on channel m up to time $k-1$, respectively. We define the belief vectors of idle and busy as: $\mathbf{b}_0(k) = [b_0(1,k), \cdots, b_0(M,k)]^T$ and $\mathbf{b}_1(k) = [b_1(1,k), \cdots, b_1(M,k)]^T$. At time k, after obtaining the sensing observations from all SUs, the belief of the channel m being idle in next time slot $k+1$ is updated at the SSDC using Bayes' formula:

$$b_0(m, k+1) = \frac{\sum_{i\in\{0,1\}} b_i(m,k) p_{i0} \prod_{n\in\mathcal{N}_m(k)} f_i(y_{m,n}(k))}{\sum_{i\in\{0,1\}} b_i(m,k) \prod_{n\in\mathcal{N}_m(k)} f_i(y_{m,n}(k))}, \tag{9}$$

where $f_i(y_{m,n}(k)) = \Pr\{y_{m,n}(k) \mid S_m(k) = i\}, \forall i \in \{0,1\}$ is the conditional pmf of SUs' observations. For the *unsensed* primary channels, the belief is updated based on the Markovian evolution of primary channels: $[b_i(m,k+1), 1-b_i(m,k+1)] = [b_i(m,k), 1-b_i(m,k)]\mathbb{P}$, where $\mathbb{P}$ is the transition matrix. The belief vectors $\mathbf{b}_0(1)$, and $\mathbf{b}_1(1)$ are initialized with the stationary distribution $\pi = [\pi_0\, \pi_1]$ of the Markov model given by $\pi = \pi\mathbb{P}$.

The reward function for channel m, at time k is defined as: $r_m(k, \mathbf{A}_k) = B_m \mathcal{I}_{\{S_m(k)=0\}} \mathcal{I}_{\{\delta_{NP}=1\}}$, where we define B_m as the bandwidth of channel m and $\mathcal{I}_E = 1$ if condition E is satisfied, and $\mathcal{I}_E = 0$ otherwise. The expected reward for channel m at time k is then given by $\mathbb{E}\{r_m(k, \mathbf{A}_k)\} = B_m b_0(m,k) P_{D,m}(k, \mathbf{A}_k)$.

We define the vector $\mathbf{S}(k) = [S_1(k), \cdots, S_M(k)] \in \mathcal{S}$ as the state of the system at time k. When the SUs do not have perfect knowledge of the states of the primary channels, the actual state of the system is the belief vector. Smallwood and Sondik have provided in [7] an algorithm to obtain the optimal decisions for this Partially Observed Markov Decision Process (POMDP) problem. With a large number of primary channels, the algorithm requires very high computational complexity and the solution is often intractable [2].

As an alternative, a *myopic* channel sensing decision can be defined to maximize the total secondary reward over all primary channels *at each time step*. The resulting sensing policy is different from the optimal POMDP solution because it does not maximize the sum of the discounted rewards accrued over time starting from each time step. That is, the myopic solution can be considered as a suboptimal solution to the POMDP problem. The *myopic* sensing decision $\mathbf{A}^*_k$ can be expressed as:

$$\mathbf{A}^*_k = \arg\max_{\mathbf{A}_k} \sum_{m=1}^{M} B_m b_0(m,k) P_{D,m}(k, \mathbf{A}_k), \tag{10}$$

subject to $\sum_{m=1}^{M} \mathbf{A}_k(m,n) = 1$, where $P_{D,m}(k, \mathbf{A}_k)$ is defined in (7). In this case, the sensing decision $\mathbf{A}^*_k$ (obtained from (10)) is the optimal solution to the myopic sensing policy, which we refer to as the *optimal myopic* solution. This solution can be obtained by listing all M^N combinations of matrix $\mathbf{A}_k$ and picking the optimal solution that maximizes (10). However, due to the complexity of this method, we propose two different methods that compute suboptimal solutions to the myopic policy and that have at most polynomial order complexities.

B. Iterative Hungarian algorithm for channel sensing policy

We propose a suboptimal algorithm for solving (10) by applying the Hungarian algorithm iteratively. For simplicity, we drop the time indices from the algorithm description and we let $B_m = 1$. We assume that the crossover probabilities of the BACs and the false alarm probability are given. We define the $M \times N$ matrix $\Delta^{(m,n)}$ such that $\Delta^{(m,n)}(m', n') = 1$ if $(m', n') = (m, n)$, and $\Delta^{(m,n)}(m', n') = 0$ otherwise. Then, we use Algorithm 2 to find the channel sensing assignment $\mathbf{A}$, which provides a suboptimal solution to (10).

We note that the complexity of the Hungarian algorithm [4] is $(\max\{M,N\})^3$ for an $M \times N$ bipartite graph. Therefore, the complexity of the proposed iterative Hungarian algorithm is in the order of $\lceil \frac{N}{M} \rceil (\max\{M,N\})^3$ since the Hungarian algorithm is computed iteratively $\lceil \frac{N}{M} \rceil$ times. In brief, the proposed algorithm can solve the sensing channel assignment with an order 4 polynomial complexity.

In particular, if $N \leq M$, Algorithm 2 is equivalent to the Hungarian algorithm which provides the *optimal* solution to (10) in this case.

Algorithm 2 Iterative Hungarian Algorithm

$\mathbf{A} = 0_{M\times N}$ and $\bar{\mathcal{N}} = \{1, \cdots, N\}$
while $\bar{\mathcal{N}} \neq \emptyset$ **do**
  $\Delta\mathbf{P} = 0_{M\times N}$
  for $m \in \{1, \cdots, M\}$ and $n \in \bar{\mathcal{N}}$ **do**
    $\Delta\mathbf{P}(m,n) = \left[P_{D,m}\left(\mathbf{A} + \Delta^{(m,n)}\right) - P_{D,m}(\mathbf{A})\right] b_0(m)$
  end for
  Run the *Hungarian algorithm* for the $M \times N$ bipartite graph whose edge weights are given in $\Delta\mathbf{P}$ to obtain the maximum sum matching.
  Remove the assigned vertices from the set $\bar{\mathcal{N}}$.
  Append the new assignments to matrix $\mathbf{A}$.
end while

C. Heuristic algorithm for channel sensing policy

We propose next a heuristic algorithm that permits to reduce the complexity to a linear order in function of the number of secondary users N. The algorithm picks randomly a secondary user n and assigns it to the channel m for which it has the highest detection probability. Also, we allow at most $\lceil \frac{N}{M} \rceil$ to sense each channel so that the SUs sense evenly all the channels and keep accurate information about the belief of every channel state. A description of the proposed heuristic sensing method is given in Algorithm 3, in which we drop the time indices for simplicity.

Algorithm 3 Heuristic Sensing Assignment

$\mathbf{A} = 0_{M\times N}$ and $\bar{\mathbb{N}} = \{1, \cdots, N\}$.
while $\bar{\mathbb{N}} \neq \emptyset$ **do**
  Pick randomly $n \in \bar{\mathbb{N}}$.
  $m^* = \arg\max_{m\in\{1,\cdots,M\}} B_m b_0(m) P_{D,m}(\Delta^{(m,n)})$
    s.t. $\sum_{n\in\{1,\cdots,N\}} \mathbf{A}(m,n) < \lceil \frac{N}{M} \rceil$
  $\mathbf{A} \leftarrow \mathbf{A} + \Delta^{(m^*,n)}$
  $\bar{\mathbb{N}} \leftarrow \bar{\mathbb{N}} \backslash n$
end while

V. Simulation Results and Discussions

We show in the simulations the average utilization of the spectrum holes as a function of the average SNR at the secondary detectors. We define the average utilization of the spectrum holes as:

$$U = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{\infty}\mathbf{B}_k(m,n)(1-S_m(k))}{\sum_{m=1}^{M}\sum_{k=1}^{\infty}(1-S_m(k))}, \quad (11)$$

where $\mathbf{B}_k(m,n)$ is the accessing decision such that $\sum_{n=1}^{N}\mathbf{B}_k(m,n) \leq 1$, meaning that at most one SU can access a channel at each time instant if it is found to be idle. The average SNR at the n-th secondary detector when sensing channel m at time k is equal to: $SNR = \frac{\sigma_x^2}{\sigma_w^2}$, and we assume that the fading coefficients $h_{m,n}(k)$ to be independent identically distributed (i.i.d.) standard Gaussian random variables. The primary channels are assumed to have independent Markovian evolutions and having the transition matrix:

$$\mathbb{P} = \begin{pmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \end{pmatrix}. \quad (12)$$

We compare the average utilization of the spectrum holes that is obtained by using the three different methods. In order to compare with the optimal myopic solution, the values of M and N are not chosen too large because, in that case, the optimal solution becomes intractable. In Fig. 3, we observe that the performance of the iterative Hungarian algorithm is close to the optimal myopic solution at low and high SNR's. However, the performance of the heuristic algorithm is close to the other two algorithms only in the low SNR region. We note that the average utilization converges to $\zeta = 0.1$ at low SNR, which conforms with the Receiver Operating Characteristics (ROC) of the Neyman-Pearson detector which becomes linear at low SNR [5], thus making the detection probability equal to the false alarm.

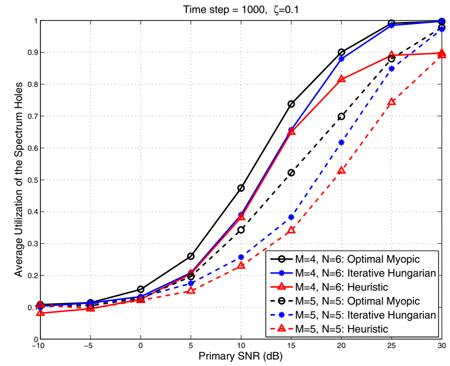

Fig. 3. Average Utilization of the spectrum holes

Note that, when $M = N = 5$, and referring to Section IV, the computational complexity is reduced by $\frac{M^N}{\lceil \frac{N}{M} \rceil (\max\{M,N\})^3} = 25$ and $\frac{M^N}{N} = 625$ times when applying the iterative Hungarian and the heuristic algorithms, respectively, as compared to the optimal myopic solution.

VI. Conclusions

In this paper, we presented a centralized spectrum sensing and accessing protocol for SUs in a CR network. We considered a more realistic CR network, compared to those that have been assumed in previous DSA designs, by taking into account the spatial and temporal variations of channel fading coefficients on the different primary channels. We derived the optimal access decision strategy and the optimal sensing decisions for a *myopic policy* assuming a centralized decision-making architecture. As an alternative to the high complexity of the optimal myopic channel sensing policy, we proposed two algorithms for obtaining near-optimal policies: The first based on the iterative Hungarian algorithm and it has fourth-order complexity while the second algorithm is based on a heuristic method and has linear complexity. The simulation results showed that the two proposed low-complexity algorithms achieve a performance very close to the optimal solution, but with a much smaller computational effort.

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