Optimal Myopic Sensing and Dynamic Spectrum Access in Centralized Secondary Cognitive Radio Networks with low-complexity Implementations

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***Abstract*—Cognitive radio (CR) techniques allow unlicensed secondary users (SUs) to opportunistically access underutilized primary channels that are licensed to primary users (PUs). We consider a multi-primary channel scenario in which the SUs cooperatively try to find these primary channel spectrum holes by limited spectrum sensing. The objective is to design the optimal sensing and accessing policy that maximizes the total secondary system throughput on the primary channels accrued over time, while satisfying a constraint on the probability of colliding with licensed transmissions. Although the problem can be formulated as a Partially Observable Markov Decision Process (POMDP), the optimal solutions are often intractable. As a result, we find the optimal *myopic* channel sensing policy that maximizes instantaneous total secondary system throughput on the primary channels at each time. The contributions of this paper include: 1) developing a universal optimal *myopic* channel sensing policy that is applicable for any number of primary channels, any number of SUs and any channel coefficients (assumed known); 2) formulation of a centralized spectrum sensing and decision-making architecture for cognitive secondary systems that allow exploitation of all available spectrum white spaces across the whole primary spectrum. We compare our combined sensing and accessing strategies with other proposed strategies and show that our proposed strategy outperforms them in terms of the resulting total secondary system throughput under the same constraints on collision with primary users.**

***Index Terms*—Cognitive radios, dynamic spectrum access, Markov chains, Neyman-Pearson detector, myopic sensing.**

I. Introduction

IT IS now widely accepted that a large number of licensed communication channels in a wide range of frequency bands are under utilized [1]. Dynamic spectrum sharing (DSS) techniques implemented on CR platforms are proposed as a method to improve the utilization of the scarce communication spectrum resources. To achieve this, though, CRs must have the ability to measure, to sense, and to learn the channel characteristics and availabilities so that they can adjust their transmission and/or reception parameters to communicate efficiently while avoiding interference with licensed and/or unlicensed users [2].

In this paper, we consider a centralized multi-primary channel scenario in which the SUs cooperatively try to find and access the primary channel spectrum holes (white spaces) by limited spectrum sensing. The objective of this problem is to design the optimal secondary system channel sensing and accessing policy that maximizes the total secondary system throughput on the primary channels accrued over time, while satisfying a constraint on the probability of colliding with licensed transmissions. Although the problem can be formulated as a Partially Observed Markov Decision Process (POMDP) problem, the optimal solutions are often intractable due to the exponential computation complexity [3]. As a result, in this paper we seek the optimal *myopic* channel sensing policy that maximizes instantaneous total secondary system throughput on the primary channels at each time, without considering the impact to future expected total secondary system throughput. We assume that the decision-making (sensing and access) in the CR communication system is centralized: a central unit gathers all channel sensing results from SUs over a dedicated control channel; the decisions of sensing and access are made at the central unit and informed to the distributed cognitive SUs over the same dedicated control channel. We call this central unit the *secondary system decision center* (SSDC). We model each primary channel occupancy dynamics as two-state (idle/busy) independent Markov chains. We assume that the state transition probabilities of the channel Markov model are known. A method provided to estimate these channel state transition probabilities can be found in [4] by formulating the primary channel sensing problem using the Hidden Markov Models (HMM). In order to compare the performance to the sensing/acess policy in [5], we consider the same assumption: secondary system has perfect knowledge of the primary signaling. Clearly, this is not always the case and indeed in many situations may not be realistic. Other sensing strategies such as waveform based sensing and cyclostationarity based sensing [2] (when partial knowledge about primary signaling is available) can easily replace the matched-filter based sensing in our model and are expected to perform no better than the matched-filter based sensing. When the secondary system has no knowledge of the primary signals, energy detector based strategies are adopted generally [2].

Many schemes presented in literature such as in [6], [3], and [5] have previously formulated the dynamic spectrum access (DSA) problem as a POMDP problem, but have left the problem unsolved because the optimal solutions are often intractable due to the exponential computation complexity. In

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all these works, suboptimal *myopic* channel sensing solutions are then proposed and derived under certain assumptions. For example, in [6] and [3], the authors developed a *myopic* channel sensing policy under the assumption of a certain ordering of Markov state transition probabilities and the assumption that the exact transition probabilities are unknown. In these two papers, the structure of accessing policies are not provided since the authors developed the *myopic* channel sensing policy based on considering the maximum expected total secondary system throughput on the primary channels at each time in their formulation. Moreover, the adopted model is not realistic because the authors assumed perfect sensing. However, in our paper, we show that the *myopic* channel sensing policy actually depends on the probability of white-space detection (channel state idle being the target). Thus we explicitly express the channel access policy based on a Neyman-Pearson detector because of the interference constraint imposed by the PUs. On the other hand, [5] assumed that all SUs are to be assigned to the single primary channel that has the highest belief of being idle at each time to sense. This model is clearly wasteful since only one primary channel can be accessed at each time no matter how many are available. This restriction reduces the total secondary system throughput because the transmission opportunities on other unsensed channels are missed entirely. On the contrary, in our model, we consider different time-varying channel fading coefficients for different SUs in modeling the nature of the wireless channels. As a result, our *myopic* channel sensing policy exploits the spatial diversity of the wireless links and makes the sensing decisions accordingly.

Moreover, our *myopic* channel sensing strategy is valid for any set of transition probabilities, any number of primary channels and/or SUs, whereas, as pointed out above, schemes of [6] and [3] are only applicable under certain conditions on channel transition probabilities. Our method is also applicable for time-varying primary channel fading coefficients, primary channels with different bandwidths and/or different signal-to-noise ratios (SNRs), although in the case of a single primary system all channel bandwidths may be identical as also assumed in the simulations in [5].

The remainder of the paper is organized as follows: In Section II we introduce the system model, SU observation model, and secondary system architecture. In Section III, the access and sensing decisions are derived. In Section IV we show simulation results using our combined access/sensing approach and compare with other existing strategies. Finally, in Section V we conclude by summarizing our results.

II. Problem Formulation

A. Primary channel state model

We use $k = \{0, 1, 2, \cdots\}$ to denote the indices of an infinite slotted time horizon. We assume a group of N SUs, and a collection of M primary channels. The primary channels are modeled as statistically identical and independent two-state (*busy* or state 1 & *idle* or state 0) Markov chains. The state *busy* indicates the channel is occupied by PUs so that it cannot be used by the SUs; the state *idle* indicates there is no PU transmissions over the channel and it is available for SUs to access. We denote the true state of primary channel $m \in \{1, \cdots, M\}$ in time slot k by $S_m(k) \in \{0, 1\}$. We assume that the state of a primary channel will not change within a single time slot. The stationary transition probability of channel m from state i to j is defined as $p_{ij} = \Pr\{S_m(k+1) = j \mid S_m(k) = i\}, \forall i, j \in \{0, 1\}$. The *transition probability matrix* of the Markov chain is denoted by $\mathbb{P} = [p_{00} \quad p_{01}; p_{10} \quad p_{11}]$.

When a SU successfully accesses a primary channel that is *actually free* during a given time slot, the SU is assumed to receive a reward proportional to the bandwidth of that channel. If a SU accesses a primary channel in state *busy*, it will cause a collision with PUs' transmission and we assume the SU gets no reward in this case. The accumulated total reward of all SUs is used as a measure of the secondary system throughput over the primary channels.

B. Secondary system decisions making architecture

In order to detect the spectrum white spaces, SUs perform channel spectrum sensing. We assume that the secondary CRs are equipped with only a single antenna. As a result, when a SU is performing channel sensing, no simultaneous communication can be performed by that SU. It is also assumed that a single SU can only sense one channel at a time. As shown in Fig. 1, SUs sense primary channels during the designated sensing periods at the beginning of each time slot. It is assumed that if a PU intends to use its channel during a transmitting period, it starts to transmit from the beginning of that time slot, so that SUs will observe a busy channel during the sensing period. On the other hand, we assume that multiple SUs can simultaneously sense the same primary channel.

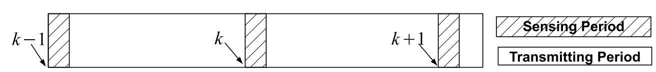

Fig. 1. Slotted time horizon with Sensing Periods and Transmitting Periods.

As mentioned before, the SSDC is assumed to collect all channel sensing results (reports) from the SUs over a dedicated control channel. We assume that the SSDC makes decisions on which SU (or a subset of SUs) should sense/access which primary channel: i.e. the secondary system decisions are centralized. We use an $M \times N$ matrix $\mathbf{A}_k$ to denote the sensing decisions made at time k, where $\mathbf{A}_k(m, n) \in \{0, 1\}, \forall m \in \{1, \cdots, M\}, n \in \{1, \cdots, N\}$. The matrix entry $\mathbf{A}_k(m, n) = 1$ or 0 stands for SU n should or should not sense primary channel m at time k, respectively. Since we assume that one SU can only sense one channel at a time, we have the constraint $\sum_{m=1}^{M} \mathbf{A}_k(m, n) = 1, \forall n \in \{1, \cdots, N\}$. Let $\mathcal{N}_m(k) = \{n \mid \forall n, \mathbf{A}_k(m, n) = 1\}$ denote the set of indices of SUs that are assigned to sense channel m during time slot k. Similarly, we use an $M \times N$ matrix $\mathbf{B}_k$ to denote the access decisions at time k, where $\mathbf{B}_k(m, n) \in \{0, 1\}, \forall m \in \{1, \cdots, M\}, n \in \{1, \cdots, N\}$. The matrix entry $\mathbf{B}_k(m, n) = 1$ or 0 stands for SU n should or should not access primary channel m at time k, respectively. We use $M \times N$ matrix $\mathbf{Y}_k$ to denote the collection of observation results from all SUs on their assigned primary channels at time k with $\mathbf{Y}_k(m, n) = y_{m,n}(k)$, where $y_{m,n}(k)$ is the report from SU n to the SSDC of the state of m-th primary user at time k. This local decision is discussed in detail in the next subsection. To make access decisions at each time k, the SSDC only need to look up the entries $\mathbf{Y}_k(m, n), \forall (m, n)$, such that

$\mathbf{A}_k(m,n) = 1$. The secondary system action procedure is summarized in Algorithm 1.

Algorithm 1 Secondary system decisions making architecture

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1. At each time k, based on previous knowledge of primary channels and channel observations, the SSDC sends out the sensing decisions $\mathbf{A}_k$ to all SUs.
2. SUs perform channel sensing according to $\mathbf{A}_k$ and sensing result $\mathbf{Y}_k$ is reported back to the SSDC.
3. Based on the channel sensing result $\mathbf{Y}_k$, SSDC send out the accessing decisions $\mathbf{B}_k$ to all SUs.
4. SUs access primary channels according to $\mathbf{B}_k$.
5. For $k \rightarrow k+1$, repeat 1 through 5.

C. Secondary user sensing models

We use $r_{m,n}(k)$ to denote the observation sample on channel m, by SU $n \in \{1, \cdots, N\}$ at time k: $r_{m,n}(k) = h_{m,n}(k)x_m(k) + w_n(k)$, where $w_n(k)$ is the zero-mean Gaussian receiver noise with variance σ_w^2 at the n-th secondary receiver (same receiver noise variance for all SUs), and $h_{m,n}(k)$ is the fading coefficient between the m-th primary transmitter and the n-th secondary receiver at time k. The channel coefficient $h_{m,n}(k)$ is assumed to be zero-mean Gaussian distributed with variance σ_h^2. We assume that the SSDC has perfect knowledge of all channel coefficients at each time k. $x_m(k)$ is assumed to be the primary signal on channel m (corresponding to the signal from m-th primary transmitter) at time k. In this paper, since we assume the secondary system has perfect knowledge of the primary signaling, we assume that the SUs use matched-filter based sensing and $x_m(k) = S_m(k) \in \{0, 1\}$.

In the CR context, when communication opportunities are scarce, it is reasonable to assume that instead of transmitting raw data $r_{m,n}(k)$'s, the SUs can only transmit quantized versions of primary channel observations as reports to the SSDC. In this paper, without loss of generality, we assume the simplest case: the reports from SUs to the SSDC are quantized to 0's and 1's.

We use $y_{m,n}(k) \in \{0, 1\}$ to denote the report of m-th primary channel state from SU n to the SSDC. Transmitting $y_{m,n}(k)$'s to SSDC are assumed to be error free. As shown in Fig. 2, the m-th channel true state $S_m(k) \in \{0, 1\}$ and the report $y_{m,n}(k) \in \{0, 1\}$ can be modeled as the input and output of a *Binary Asymmetric Channel* (BAC), respectively. The two hypotheses on channel m are $\mathcal{H}_1 : S_m(k) = 0$

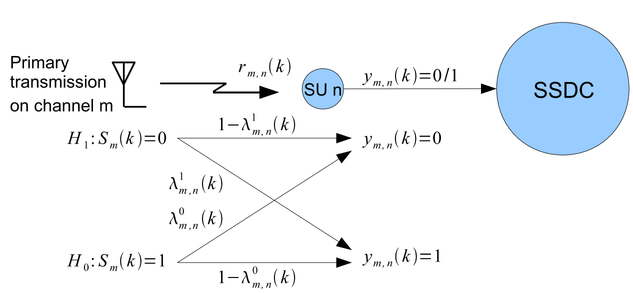

Fig. 2. SUs' reports of observations on primary channels can be modeled as Binary Asymmetric Channels.

and $\mathcal{H}_0 : S_m(k) = 1$, respectively. We use $\lambda_k^1(m,n)$, and $\lambda_k^0(m,n)$ to denote the crossover probability under $\mathcal{H}_1$, and $\mathcal{H}_0$, respectively. We assume $y_{m,n}(k)$ is quantized based on the *maximum a posteriori probability* (MAP) detector:

1) if $h_{m,n}(k) > 0$,

$$y_{m,n}(k) = \begin{cases} 0 & \text{if } r_k(m,n) \leq \eta'_{m,n}(k), \\ 1 & \text{if } r_k(m,n) > \eta'_{m,n}(k). \end{cases}$$

2) if $h_{m,n}(k) < 0$,

$$y_{m,n}(k) = \begin{cases} 0 & \text{if } r_k(m,n) \geq \eta'_{m,n}(k), \\ 1 & \text{if } r_k(m,n) < \eta'_{m,n}(k). \end{cases}$$

where $\eta'_{m,n}(k) = \frac{h_{m,n}(k)^2 - 2\log(\eta_m(k))\sigma_w^2}{2h_{m,n}(k)}$ and $\eta_m(k) = \frac{\Pr\{S_m(k)=1\}}{\Pr\{S_m(k)=0\}}$. The resulting crossover probabilities can be expressed as: 1) when $h_{m,n}(k) > 0$, $\lambda^1_{m,n}(k) = Q\left(\frac{\eta'_{m,n}(k)}{\sigma_w}\right)$ and $\lambda^0_{m,n}(k) = Q\left(\frac{h_{m,n}(k) - \eta'_{m,n}(k)}{\sigma_w}\right)$; 2) when $h_{m,n}(k) < 0$, $\lambda^1_{m,n}(k) = Q\left(\frac{-\eta'_{m,n}(k)}{\sigma_w}\right)$ and $\lambda^0_{m,n}(k) = Q\left(\frac{\eta'_{m,n}(k) - h_{m,n}(k)}{\sigma_w}\right)$.

III. Access and Sensing Decisions

A. Access decisions at the SSDC

To meet the requirement of keeping the collision probability with PUs on every channel under a given constraint, the optimal access-decisions at the SSDC must be based on a Neyman-Pearson detector [7] at the SSDC as the access decision rule. For simplicity, we use the variable length vector $\mathbf{y}_k(m,:) = \{y_{m,n}(k) : \forall n \in \mathcal{N}_m(k)\}$ to denote all channel sensing reports at time k, from the SUs on channel m and the variable length vector $\mathbf{y}_{0:k}(m,:) = \{\mathbf{y}_0(m,:), \cdots, \mathbf{y}_k(m,:)\}$ to denote the sensing history on channel m, from time 0 to k.

At time k, for the m-th primary channel, the SSDC chooses one of the two possible hypotheses based on $\mathbf{y}_{0:k}(m,:)$:

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{y}_{0:k}(m,:) \sim P_{m,1} \text{ (channel } \textit{idle}\text{)} \\ \mathcal{H}_0 &: \mathbf{y}_{0:k}(m,:) \sim P_{m,0} \text{ (channel } \textit{busy}\text{)}, \end{aligned}$$

where $P_{m,1}$, and $P_{m,0}$ denote the conditional probability density of the vector $\mathbf{y}_{0:k}(m,:)$ given $S_m(k) = 0$, and $S_m(k) = 1$, respectively. The corresponding likelihood ratio detector based on $\mathbf{y}_{0:k}(m,:)$ is generally complicated and hard to derive in closed form exactly due to the fact that at each time k, the number of SUs that are on channel m changes and as time evolves, the complexity increases. To simplify the access decision structure, we assume that the access decisions are made based only on the current observations $\mathbf{y}_k(m,:)$. Then, for the m-th primary channel, the likelihood ratio is defined as $\mathcal{L}(\mathbf{y}_k(m,:)) = \frac{P_{m,1}(\mathbf{y}_k(m,:))}{P_{m,0}(\mathbf{y}_k(m,:))}$, where we reuse the notations $P_{m,1}$, and $P_{m,0}$ to denote the conditional probability density of vector $\mathbf{y}_k(m,:)$ given $S_m(k) = 0$, and $S_m(k) = 1$, respectively.

The corresponding Log-likelihood ratio is given by $\mathcal{LLR}(\mathbf{y}_k(m,:)) = \sum_{n \in \mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k) + d_m(k)$, where we define $c_{m,n}(k) = \ln\left(\frac{\lambda^1_{m,n}(k)}{1-\lambda^0_{m,n}(k)} \cdot \frac{\lambda^0_{m,n}(k)}{1-\lambda^1_{m,n}(k)}\right)$, and $d_m(k) = \sum_{n \in \mathcal{N}_m(k)} \ln\left(\frac{1-\lambda^1_{m,n}(k)}{\lambda^0_{m,n}(k)}\right)$. A sufficient statistic is $\sum_{n \in \mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k)$, i.e., the test

$\mathcal{LLR}(\mathcal{Y}_k(m,:)) \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \tau_m(k)$ is equivalent to the test $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k) \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \tau_m(k) - d_m(k) = \tau'_m(k)$.

We use $f^i_{m,k}$, and $F^i_{m,k}$ to denote the conditional probability mass function (pmf), and the conditional cumulative distribution function (cdf) of random variable $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k)$ under hypothesis $\mathcal{H}_i$, respectively.

Let ζ denote the collision probability constraint on each individual primary channel. Then $\tau'_m(k)$ is chosen such that: $1 - F^0_{m,k}(\tau'_m(k)) \le \zeta < 1 - F^0_{m,k}(\tau'_m(k)+1)$. The randomized access decision rule is then given by

$$\tilde{\delta}_{NP}(\mathcal{Y}_k(m,:)) = \begin{cases} 1 & \text{if } \sum y_{m,n}(k)c_{m,n}(k) > \tau'_m(k) \\ \gamma_m(k) & \text{if } \sum y_{m,n}(k)c_{m,n}(k) = \tau'_m(k) \\ 0 & \text{if } \sum y_{m,n}(k)c_{m,n}(k) < \tau'_m(k) \end{cases},$$

where the decisions 1, and 0 stand for access and do not access, respectively; and when $\sum_{n\in\mathcal{N}_m(k)} y_{m,n}(k)c_{m,n}(k) = \tau'_m(k)$ access with probability $\gamma_m(k)$. Therefore, the SSDC decides to access channel m only if $\delta_{NP}(\mathcal{Y}_k(m,:)) = 1$, where δ_{NP} is a binomial random variable with a probability of success equal to $\tilde{\delta}_{NP}$. The randomization variable is given by $\gamma_m(k) = \frac{\zeta - F^0_{m,k}(\tau'_m(k))}{F^0_{m,k}(\tau'_m(k)-1) - F^0_{m,k}(\tau'_m(k))}$. The probability of correctly detecting white spaces is then found by the following (notice that $\mathbf{A}_k$ defines $\mathcal{N}_m(k), \forall m$):

$$\begin{aligned} & P_{D,m}(k,\mathbf{A}_k) = \Pr\{\delta_{NP} = 1 \mid \mathcal{H}_1\} \\ = \; & 1 - F^1_{m,k}(\tau'_m(k)) + \gamma_m(k)\cdot f^1_{m,k}(\tau'_m(k)). \end{aligned} \quad (1)$$

This probability of detection is used in the sensing decision making at the SSDC as described in the next section.

B. Optimal and myopic sensing decisions at the SSDC

The objective of designing the sensing decision rule is to maximize the total secondary system reward on all channels accrued over time. To do this, let's first define $b_0(m,k) = \Pr\{S_m(k) = 0 \mid \mathbf{y}_{0:k-1}(m,:)\}$, and $b_1(m,k) = 1 - b_0(m,k)$ as the belief of channel m being *idle* and *busy* at time k, given the observation history on channel m up to time $k-1$, respectively. We define the belief vectors of idle and busy as: $\mathbf{b}_0(k) = [b_0(1,k),\cdots,b_0(M,k)]^T$ and $\mathbf{b}_1(k) = [b_1(1,k),\cdots,b_1(M,k)]^T$. At time k, after obtaining sensing observations from all SUs, the belief of the channel m being idle in next time slot $k+1$ is updated at the SSDC using Bayes' formula: $b_0(m,k+1) = \frac{\sum_{i\in\{0,1\}} p_{i0}\left[\prod_{n\in\mathcal{N}_m(k)} f_i(y_{m,n}(k))\right] b_i(m,k)}{\sum_{i\in\{0,1\}}\left[\prod_{n\in\mathcal{N}_m(k)} f_i(y_{m,n}(k))\right] b_i(m,k)}$, where we denote $f_i(y_{m,n}(k)) = \Pr\{y_{m,n}(k) \mid S_m(k) = i\}, \forall i \in \{0,1\}$ as the conditional pmf of SUs' reports. For those *unsensed* primary channels, the belief is updated based on the Markovian evolution of primary channels: $[b_i(m,k+1), 1-b_i(m,k+1)] = [b_i(m,k), 1-b_i(m,k)]\mathbb{P}$, where $\mathbb{P}$ is the transition matrix. The belief vectors $\mathbf{b}_0(1)$, and $\mathbf{b}_1(1)$ are initialized with the stationary distribution $\pi = [\pi_0\, \pi_1]$ of the Markov model given by $\pi = \pi\mathbb{P}$.

The reward function for channel m, at time k is defined as: $r_m(k,\mathbf{A}_k) = B_m \mathcal{I}_{\{S_m(k)=0\}}\mathcal{I}_{\{\delta_{NP}=1\}}$, where we define B_m as the bandwidth of channel m and $\mathcal{I}_E$ is the indicator function of event E. The expected reward for channel m at time k is then given by $\mathbb{E}\{r_m(k,\mathbf{A}_k)\} = B_m b_0(m,k)P_{D,m}(k,\mathbf{A}_k)$, where $P_{D,m}(k,\mathbf{A}_k)$ is defined in (1).

We define the vector $\mathbf{S}(k) = [S_1(k),\cdots,S_M(k)] \in \mathcal{S}$ as the state of the system at time k. When the SUs do not have perfect knowledge of the states of the primary channels, the actual state of the system is the belief vector. Smallwood and Sondik have provided in [8] an algorithm to obtain the optimal decisions for this POMDP problem. However, when the number of primary channels is large, the algorithm requires very high computational complexity and the solution is often intractable [3].

As an alternative, an optimal *myopic* channel sensing decision can be defined to maximize the total secondary reward over all primary channels *at each time step*. This *myopic* sensing decision $\mathbf{A}^*_k$ can be expressed as:

$$\mathbf{A}^*_k = \arg\max_{\mathbf{A}_k} \sum_{m=1}^{M} B_m b_0(m,k) P_{D,m}(k,\mathbf{A}_k), \quad (2)$$

under constraint $\sum_{m=1}^{M} \mathbf{A}_k(m,n) = 1$.

Because the entries of the matrix $\mathbf{A}^*_k$ can only have value 0 and 1, the objective function in (2) is nonlinear, and we have the constraint $\sum_{m=1}^{M} \mathbf{A}_k(m,n) = 1$, the above optimization problem can be cast as a *constrained nonlinear 0-1 programming* problem [9]. Since the objective function is non-separable, the solution is generally hard to find. To find the optimal solution $\mathbf{A}^*_k$, one method is the direct search which has an exponential complexity of M^N.

As an alternative with much lower computational complexity, we propose a suboptimal algorithm for solving (2) by using a Hungarian algorithm [10] iteratively. For simplicity, we drop the time indices from the algorithm description and we let $B_m = 1$. We assume that the crossover probabilities of the BACs and the false alarm probability are given. We define the $M \times N$ matrix $\Delta^{(m,n)}$ such that $\Delta^{(m,n)}(m',n') = 1$ if $(m',n') = (m,n)$, and $\Delta^{(m,n)}(m',n') = 0$ otherwise. Then, we use Algorithm 2 to find the channel sensing assignment $\mathbf{A}$, which provides a suboptimal solution to (2).

We note that the complexity of the Hungarian algorithm is $(\max\{M,N\})^3$ for an $M \times N$ bipartite graph. Therefore, the complexity of the proposed iterative Hungarian algorithm is in the order of $\lceil \frac{N}{M} \rceil (\max\{M,N\})^3$ since the Hungarian algorithm is computed iteratively $\lceil \frac{N}{M} \rceil$ times. In brief, the proposed algorithm can solve the sensing channel assignment with roughly an order 4 polynomial complexity.

In particular, if $N \le M$, Algorithm 2 is equivalent to the Hungarian algorithm which provides the *optimal* solution to (2) in this case.

Algorithm 2 Iterative Hungarian Algorithm

$\mathbf{A} = 0_{M\times N}$ and $\bar{\mathcal{N}} = \{1,\cdots,N\}$
while $\bar{\mathcal{N}} \neq \emptyset$ **do**
  $\Delta\mathbf{P} = 0_{M\times N}$
  for $m \in \{1,\cdots,M\}$ and $n \in \bar{\mathcal{N}}$ **do**
    $\Delta\mathbf{P}(m,n) = \left[P_{D,m}\left(\mathbf{A} + \Delta^{(m,n)}\right) - P_{D,m}(\mathbf{A})\right] b_0(m)$
  end for
  Run the *Hungarian algorithm* for the $M \times N$ bipartite graph whose edge weights are given in $\Delta\mathbf{P}$ to obtain the maximum sum matching.
  Remove the assigned vertices from the set $\bar{\mathcal{N}}$.
  Append the new assignments to matrix $\mathbf{A}$.
end while

IV. Simulation Results and Discussions

In this section, we compare the performance of our proposed sensing/access decisions with those proposed in [5], in which at each time, all SUs sense one single primary channel with the highest belief of being idle.

In order to directly compare the performance of our proposed optimal *myopic* sensing solution with the results of [5], we firstly simulate the discounted secondary system reward and we make the same exact assumptions made in [5]: 1) a discount factor 0.999 is assumed for time horizon from 0 to 10000; 2) the SUs' sensing reports to the SSDC are directly the observations $r_{m,n}(k)$'s; 3) all channel coefficients $h_{m,n}(k)$'s are set to 1's for all time; 4) the transition probability matrix of the Markov channel model: $\mathbb{P} = [0.9 \quad 0.1; 0.2 \quad 0.8]$; 5) unit bandwidth for all primary channels; 6) the constraint on the probability of collisions with PUs is $\zeta = 0.1$.

In Fig. 3, we show the simulation results of discounted reward based on 2 primary channels, with single SU and also 2 SUs, for $SNR_{dB} = 20\log_{10}(\frac{1}{\sigma_w^2})$, from -5 to $5dB$. The performance of the approach proposed in [5] is exactly regenerated in this figure (2 primary channels, 1 SU). Since when there is only a single SU, the two strategies are equivalent, we see that the optimal allocation of SUs and the approach in [5] give the same discounted reward. However, when there are 2 SUs (the rest of assumptions staying the same), we see that our proposed approach leads to a higher discounted reward. This is because when all SUs are allocated to sense a single channel, SUs lose access opportunities on the other channel.

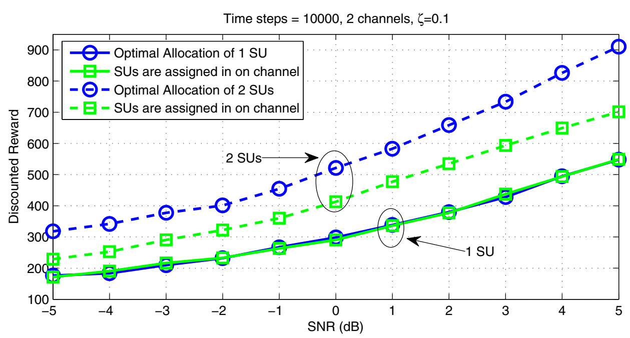

Fig. 3. Discounted reward comparison between our proposed method and the method proposed by [5]

Next, we compare the resulting *percentage of channel usage* (the percentage of successful access out of total white spaces of all primary channels) of our proposed sensing/access strategy (direct-search optimal solution and solution found by using iterative Hungarian algorithm) to the one in [5] under following assumptions: 1) no discount factor; 2) SUs' channel sensing reports are based on the MAP detector for both competitive strategies (reports are 0's and 1's); 3)transition probability matrix of the Markov channel model: $\mathbb{P} = [0.9 \quad 0.1; 0.2 \quad 0.8]$; 4) channel coefficients are standard Gaussian distributed: $h_{m,n}(k) \sim \mathcal{N}(0, 1)$ and known at the SSDC at each time; 5) unit bandwidth for all primary channels; 5) constraint on the probability of collisions with PUs is $\zeta = 0.1$. As shown in Fig. 4, we consider two scenarios: 1) 2 primary channels and 3 SUs; 2) 3 primary channels and 11 SUs (without showing optimal solution due to high computational complexity). We can see that under both conditions, our proposed approach outperforms the approach in [5]. We can also see that when the ratio of the number of SUs to the number of primary channels is higher, our proposed strategy leads to better percentage of channel usage; whereas, the approach used in [5] gives worse results when the ratio of the number of SUs over the number of primary channels goes higher. Indeed, since the approach used in [5] allocates all the SUs to only one channel at every time (the channel with the highest belief of being idle), the higher the aforementioned ratio is, the more transmission opportunities on primary channels are wasted.

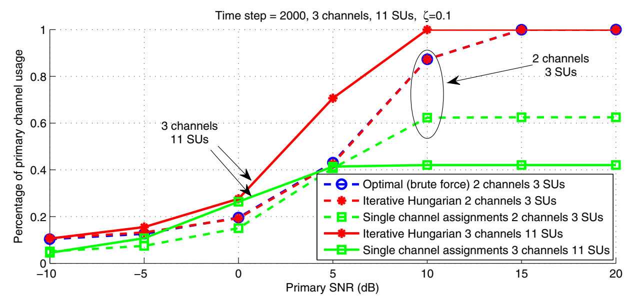

Fig. 4. Percentage of primary channel usage comparing: 1) brute force optimal solution; 2) iterative Hungarian algorithm; 3) all SUs on a single channel at a time [5].

V. Conclusions

We proposed and established a universal optimal *myopic* channel sensing policy in the case of a centralized Cognitive Radio Communication System in which the channel sensing and access decisions are made at a central unit. Unlike other existing approaches proposed in the literature, our universal optimal *myopic* channel sensing policy is more realistic because our policy solves who-goes-to-where problem and we introduced the channel access structure dependency explicitly (applying the Neyman-Pearson detector as the channel access decision rule at the SSDC). We also showed that our approach outperforms existing/possible approaches.

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