Efficient Spectrum Sharing with Autonomous Primary Users: Distributed Dynamic Spectrum Leasing (D-DSL)

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***Abstract*—In this paper we introduce a new architecture for dynamic spectrum sharing called the distributed dynamic spectrum leasing (D-DSL) and a game theoretic framework for its implementation on a cognitive radio network. In D-DSL, it is assumed that each channel is assigned to a primary user who may lease the channel to secondary users. The spectrum owners can thus dynamically adjust the amount of interference they are willing to tolerate from the secondary users. On the other hand, the secondary users compete with each other in order to achieve maximum possible Quality of Service without violating the interference cap imposed by the primary users on each channel. The secondary users are allowed to transmit in more than one channel simultaneously. Under these conditions in addition to the competition among secondary users, the primary users also compete with each other to let more secondary users to access their channels. We establish conditions for the system to reach an equilibrium and analyze the performance. It is shown that the performance of the secondary system improves by increasing the availability of primary channels.**

I. INTRODUCTION

The rapid growth of wireless communication has resulted in an increasing demand for the wireless bandwidth. On the other hand some allocated spectrum bands have found to be underutilized as noted in several recent studies by the Federal Communications Commission (FCC) [1]. This has led the FCC to allow unlicensed wireless users to access the licensed spectrum bands under the concept of spectrum sharing. The so-called cognitive radio has positioned the dynamic spectrum sharing as a realistic technology over the last several years [2].

Under the Dynamic Spectrum Sharing (DSS), the spectrum owner allows the unlicensed users to dynamically access its spectrum. In almost all existing proposals, the secondary users are solely responsible for the interference management and coexistence in the primary spectrum band. These proposals are termed as Dynamic Spectrum Access (DSA). In [3], they investigate the fairness and efficiency of the DSA systems by imposing punishment strategies.

Recently in [4]–[6], the authors introduced the concept of Dynamic Spectrum Leasing (DSL) in which the primary users, as well as the secondary users, are involved in managing the interference. In DSL, each secondary user acts selfishly to fully utilize the primary spectrum band. In [7], the authors generalized their earlier framework by considering several primary users in the system. As shown in Fig. 1(a), a central unit measure the total interference I_0 from all secondary users and sets a *common* interference cap (IC), denoted by Q_0, that is valid for all primary users in the channel. The IC is the maximum interference that the primary system is willing to tolerate from all secondary transmissions. This framework is termed as Centralized-DSL due to the assumed central coordination among primary users.

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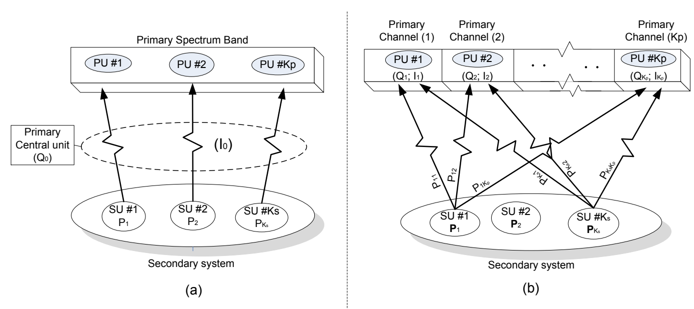


Fig. 1. a) The Centralized Dynamic Spectrum Leasing (C-DSL). b) The Distributed Dynamic Spectrum Leasin (D-DSL)

In this paper, on the other hand, we introduce a novel framework called the Distributed Dynamic Spectrum Leasing (D-DSL). In contrast to C-DSL, each primary user in a D-DSL system sets its own IC depending on the interference level from the secondary users. The secondary users are autonomous agents that are allowed to transmit simultaneously in more than one spectrum band in order to capitalize on, and fully utilize, the available spectrum opportunities. We model this scenario as a spectrum sharing game. The non-cooperative game in this new framework will not only be between the primary and the secondary users as in C-DSL, but also be a non-cooperative game among primary users themselves.

This paper develops a game-theoretic framework for such a D-DSL system by identifying suitable payoff functions for both primary and secondary users, and establishes conditions for existence of an equilibrium point that can be reached via adaptive best-response of users. As in previous work on DSL, the proposed D-DSL can be implemented with the same inter-system control information exchanges assumed in [4]–[6]. We study the system performance as a function of the secondary system size and number of primary channels. Using simulations we evaluate the performance of the proposed D-DSL and we show the improvement that the secondary system rate achieves by increasing the number of degrees of freedom.

The remainder of this paper is organized as follows: Section II describes the D-DSL framework and the system model. Section III presents the game theoretic formulation. Section IV discusses the existence and uniqueness of a Nash equilibrium. In section V, we evaluate the performance of a spectrum sharing network based on the proposed D-DSL. Finally Section VI concludes the paper.

II. D-DSL - Based Cognitive Radio System Model

We assume that there are K_p channels each of them licensed to a primary system. Without loss of generality, we assume that there is one primary transmitter receiver pair in every channel. There are K_s secondary transmitter-receiver pairs (links) that are active on the primary channels. In this paper we introduce the concept of Distributed Dynamic Spectrum Leasing (D-DSL) in which the primary user in the j-th channel, for $j \in \mathcal{K}_p$, measures the total secondary interference I_j on its own channel and sets its own interference cap, denoted by Q_j, that is applicable for only the j-th channel. As shown in Fig. 1(b), each secondary user now has the opportunity to communicate over multiple primary channels by allocating its transmit power appropriately for different primary channels. We denote the k-th secondary user's power vector by $\mathbf{p}_k = \left(p_{k,1}, p_{k,2}, \cdots, p_{k,K_p}\right)^T$, for $k \in \mathcal{K}_s$, where $p_{k,j}$ is the power it allocates to communicate over the j-th primary channel.

When a primary user raises its interference cap, it encourages secondary users to prefer that particular channel over other primary channels. This leads to a non-cooperative game among primary users in which the reward of a primary user should be an increasing function of its interference cap. However, each primary user should maintain a target signal-to-interference-plus-noise-ratio (SINR) to ensure its required QoS. Each secondary user is assumed to act selfishly to maximize its own total utility. However, the transmission powers $p_{k,j}$'s should be carefully controlled to maintain an overall interference level I_j that is below the primary interference cap Q_j for all $j \in \mathcal{K}_p$. As one would expect, in a D-DSL network, the secondary system has more flexibility compared to that in a C-DSL based network considered in [7].

For each $j \in \mathcal{K}_p$, the primary user in the j-th channel will be labeled by 0 and the secondary links (transmitter-receivers pairs) are labeled from 1 through K_s. The set of user indices (primary and secondary) in every channel is denoted by $\mathcal{K}_c$, i.e. $\mathcal{K}_c = \{0\} \cup \mathcal{K}_s$. The channel gain between the k-th receiver in the j-th channel and k'-th transmitter is denoted by $h_{k,k'}^{(j)}$ for $k, k' \in \mathcal{K}_c$ and $j \in \mathcal{K}_p$ and let $A_{k,k'}^{(j)} = h_{k,k'}^{(j)} \sqrt{p_{k',j}}$ be the received signal amplitude. As in [4], we may obtain a discrete-time representation of the received signal at the primary receiver in the j-th channel as

$$\mathbf{r}_{0,j} = A_{0,0}^{(j)} b_{0,j} \mathbf{s}_0^{(p)} + \sum_{k \in \mathcal{K}_s} A_{0,k}^{(j)} b_{k,j} \mathbf{s}_k^{(p)} + \sigma_{0,j} \mathbf{n}_{0,j}, \qquad (1)$$

where the vectors $\mathbf{r}_{0,j} = \left(r_{0,1}^{(j)}, \cdots, r_{0,M}^{(j)}\right)^T$ and $\mathbf{s}_k^{(p)} = \left(s_{k,1}^{(p)}, \cdots, s_{k,M}^{(p)}\right)$ are the vector representations of the primary received signal and signal constellations, respectively, with respect to an M-dimensional primary basis and $\mathbf{n}_{0,j} \sim \mathcal{N}(\mathbf{0}, \mathbf{I_M})$. Similarly, a discrete-time representation of the received signals at the k-th secondary receiver for $k \in \mathcal{K}_s$ in the j-th channel is

$$\mathbf{r}_{k,j} = \sum_{k' \in \mathcal{K}_s} A_{k,k'}^{(j)} b_{k',j} \mathbf{s}_{k'}^{(s)} + A_{k,0}^{(j)} b_{0,j} \mathbf{s}_0^{(s)} + \sigma_{k,j} \mathbf{n}_{k,j} \qquad (2)$$

where the vectors $\mathbf{r}_{k,j} = \left(r_{k,1}^{(j)}, \cdots, r_{k,N}^{(j)}\right)^T$ and $\mathbf{s}_k^{(s)} = \left(s_{k,1}^{(s)}, \cdots, s_{k,M}^{(s)}\right)$ are the vector representations of the secondary received signal and the signal constellation, respectively, with respect to the N-dimensional secondary basis and $\mathbf{n}_{k,j} \sim \mathcal{N}(\mathbf{0}, \mathbf{I_N})$.

In the following we assume that all transmissions are modulated as binary phase shift keying (BPSK), and the detectors are based on the matched filter (MF) outputs. Therefore the primary decisions in the j-th channel are given by $\hat{b}_{0,j} = \text{sgn}\left(y_{0,j}\right)$ where $y_{0,j} = A_{0,0}^{(j)} b_{0,j} + \sum_{k \in \mathcal{K}_s} \rho_{0k}^{(p)} A_{0,k}^{(j)} b_{k,j} + \sigma_{0,j} \eta_{0,j}$, with $\rho_{0k}^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{s}_k^{(p)}$ and $\eta_{0,j} \sim \mathcal{N}(0,1)$. The total secondary interference I_j from all secondary transmissions to the primary-user in the j-th channel is $I_j = \sum_{k \in \mathcal{K}_s} \left(\rho_{0k}^{(p)} A_{0,k}^{(j)}\right)^2$. Similarly, the k-th secondary link estimates its symbols on the j-th channel as $\hat{b}_{k,j} = \text{sgn}\left(y_{k,j}\right)$, where $y_{k,j} = A_{k,k}^{(j)} b_{k,j} + \sum_{k' \in \mathcal{K}_c \setminus k} \rho_{k,k'}^{(s)} A_{k,k'}^{(j)} b_{k',j} + \sigma_{k,j} \eta_{k,j}$ where $\rho_{k,k'}^{(s)} = \left(\mathbf{s}_k^{(s)}\right)^T \mathbf{s}_{k'}^{(s)}$ for $k' \in \mathcal{K}_c$ and $\eta_{k,j} \sim \mathcal{N}(0,1)$.

III. A Game Model for D-DSL

In the proposed D-DSL game model, the users interact with each other by adjusting the tolerable interference caps of the primary users and the transmit powers of the secondary users in each channel. We model the D-DSL system as the following non-cooperative game $G = \left(\mathcal{K}, \mathcal{A}_{(.,.)}, u_{(.,.)}\right)$.

1) Players: the player set $\mathcal{K} = \mathcal{K}_s \cup \mathcal{K}_p$. We represent the secondary players by the index k for $k \in \mathcal{K}_s$ and the primary players by j for $j \in \mathcal{K}_p$.
2) Action Space: $\mathcal{P} = \mathcal{A}_{s,1} \times \mathcal{A}_{s,2} \times \cdots \times \mathcal{A}_{s,K_s} \times \mathcal{A}_{p,1} \cdots \times \mathcal{A}_{p,K_p}$, where $\mathcal{A}_{s,k} = \mathcal{P}_k = \mathcal{P}_{k,1} \times \cdots \times \mathcal{P}_{k,K_p}$ for $k \in \mathcal{K}_s$ represents the action space of the k-th secondary player with $\mathcal{P}_{k,j} = [0, \overline{P}_k]$ and $\mathcal{A}_{p,j} = \mathcal{Q}_j = [0, \overline{Q}_j]$ for $j \in \mathcal{K}_p$ represents the action set of the j-th primary user. The upper limits of the action

sets $\overline{P}_k$ and $\overline{Q}_j$ represent, respectively, the maximum transmission power of the k-th secondary user and the maximum tolerable interference cap of the j-th primary user. We denote the action vector of all users by $\mathbf{a} = (\mathbf{p}_1, \cdots, \mathbf{p}_{K_s}, Q_1, \cdots, Q_{K_p})^T$ where $Q_j \in \mathcal{Q}_j$ for $j \in \mathcal{K}_p$ and $\mathbf{p}_k = (p_{k,1}, \cdots, p_{k,K_p})^T$ is the action set of the k-th secondary user for $k \in \mathcal{K}_s$ with $p_{k,j} \in \mathcal{P}_{k,j}$ and $\|\mathbf{p}_k\| = \sum_{j \in \mathcal{K}_p} p_{k,j} \leq \overline{P}_k$. For notational convenience, we refer to the action vector excluding that of the k-th secondary player by $\mathbf{a}_{-s,k}$ for $k \in \mathcal{K}_s$ and the action vector excluding the j-th primary player by $\mathbf{a}_{-p,j}$ for $j \in \mathcal{K}_p$.

3) Utility function: We denote by $u_{s,k}(\mathbf{p}_k, \mathbf{a}_{-s,k})$, for $k \in \mathcal{K}_s$, the k-th secondary user's utility function, and by $u_{p,j}(Q_j, \mathbf{a}_{-p,j})$ for $j \in \mathcal{K}_p$, the j-th primary user's utility function.

At any given time t, the *assumed* worst-case target SINR of the j-th primary user is defined as follows:

$$\overline{\gamma}_{0,j} = \frac{\left(h_{0,0}^{(j)}\right)^2 p_{0,j}}{Q_j + \sigma_{0,j}^2}. \tag{3}$$

The instantaneous SINR of the j-th primary user, on the other hand, is $\gamma_{0,j} = \frac{\left(h_{0,0}^{(j)}\right)^2 p_{0,j}}{\sum_{k \in \mathcal{K}_s} \left(\rho_{0k}^{(p)} h_{0k}^{(j)}\right)^2 p_{k,j} + \sigma_{0,j}^2}$. We choose the utility function of the j-th primary user for $j \in \mathcal{K}_p$ to be [4]:

$$u_{p,j}\left(Q_j, \mathbf{a}_{-p,j}\right) = \left(\overline{Q}_j - \left(Q_j - I_j\left(\mathbf{a}_{-p,j}\right)\right)\right) Q_j. \tag{4}$$

The SINR of the k-th secondary user in the j-th channel is

$$\gamma_{k,j} = \frac{\left|h_{k,k}^{(j)}\right|^2 p_{k,j}}{\sum_{k' \in \mathcal{K}_c \setminus k} \left(\rho_{k,k'}^{(s)}\right)^2 \left|h_{k,k'}^{(j)}\right|^2 p_{k',j} + \sigma_{k,j}^2}. \tag{5}$$

A reasonable utility function for the k-th secondary user should be a monotonically increasing function of the SINR $\gamma_{k,j}$ for $\forall j \in \mathcal{K}_p$. At the same time it should be a decaying function of $I_j - Q_j$ for every $j \in \mathcal{K}_p$. Hence, we propose the following secondary user utility function, for $k \in \mathcal{K}s$:

$$\begin{aligned} u_{s,k}\left(\mathbf{p}_k, \mathbf{a}_{-s,k}\right) &= \sum_{j \in \mathcal{K}_p} u_{k,j}(p_{k,j}) \\ &= \sum_{j \in \mathcal{K}_p} \left(Q_j - \lambda_j I_j\right) f_{k,j}\left(p_{k,j}\right), \end{aligned} \tag{6}$$

where $u_{k,j}(p_{k,j})$ is the partial utility that the k-th user obtains by transmitting on the j-th channel, λ_j is a positive coefficient which controls how strictly the secondary users need to obey the j-th primary interference cap Q_j and $f_{k,j}(.)$ is the reward function of the k-th secondary user on j-th channel which should typically be an increasing function of $p_{k,j}$.

IV. The Existence of a Nash Equilibrium in the proposed D-DSL game

In this section we investigate the existence and uniqueness of an equilibrium in the above D-DSL game $G = \left(\mathcal{K}, \mathcal{A}_{(.,.)}, u_{(.,.)}\right)$. The Nash equilibrium is a predictable and stable outcome for the non-cooperative D-DSL game in which no user can benefit by changing its action while the other users keep theirs fixed [8]. But such a point may not necessarily exist in a game.

Proposition 1: There exists a Nash equilibrium for the D-DSL game $G = (\mathcal{K}, \mathcal{A}_k, u_k)$ if the action space $\mathcal{A}_k$ is a nonempty compact convex subsets of an Euclidian space $\mathbb{R}^n$ for all $k \in \mathcal{K}$ and the primary-secondary utility functions are continuous in $\mathbf{a}$ and quasi-concave in a_k [8].

Proof: Clearly the action spaces of primary and secondary users are compact convex nonempty sets. The primary utility function $u_{p,j}\left(Q_j, \mathbf{a}_{-p,j}\right)$ is continuous in $\mathbf{a}$ and concave in Q_j. The secondary utility function $u_{s,k}\left(\mathbf{p}_k, \mathbf{a}_{-s,k}\right)$ is continuous in the action vector $\mathbf{a}$ and it is concave if the partial utility functions $u_{k,j}(p_{k,j})$ for $j \in \mathcal{K}_p$ are concave in $p_{k,j}$. Thus all the necessary conditions in proposition 1 are satisfied and ensuring the existence of a Nash equilibrium. ■

In dynamic spectrum leasing the goal of each secondary user can be considered to be to maximize the rate it can achieve. To that end, we will set the reward function in (6) to be $f_{k,j}(p_{k,j}) = W_j \log\left(1 + \gamma_{k,j}\right)$ where $\gamma_{k,j}$ is the k-th secondary user's received SINR on the j-th channel as defined in (5) and W_j is a positive weighting coefficient which can be taken to be proportional to the bandwidth of channel j. Note that this reward function $f_{k,j}(p_{k,j})$ is an increasing concave function which satisfies the conditions for the existence of a Nash equilibrium.

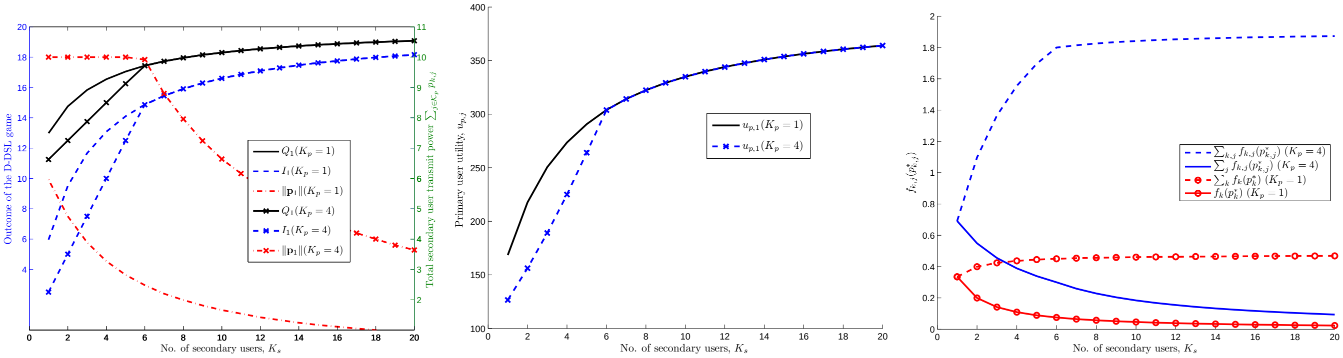
The best response of a particular player in a non-cooperative is the reaction that maximizes its own utility for a fixed action vector of the other players:

Definition 1: The best response correspondence of the k-th player $r_k : \mathcal{A}_{-k} \rightarrow \mathcal{A}_k$ is the set $r_k(\mathbf{a}_{-k}) = \{a_k \in \mathcal{A}_k : u_k(a_k; \mathbf{a}_{-k}) \geq u_k(a_k'; \mathbf{a}_{-k})$ for all $a_k' \in \mathcal{A}_k\}$.

The best response of the primary user in a D-DSL game is the *unique*[1] Q_j^* that maximizes its utility $u_{p,j}(Q_j)$. It can be shown that $Q_j^*\left(I_j\right) = \frac{\overline{Q}_j + I_j}{2}$ for all $j \in \mathcal{K}_p$. Since the primary utility is an increasing function when $Q_j \leq Q_j^*$, if Q_j^* exceeds the maximum interference cap $\overline{Q}_j$, the j-th primary user will set its interference cap to be $Q_j = \overline{Q}_j$. Therefore the best response of the j-th primary user is $r_{p,j}(a_{-p,j}) = \min\{Q_j^*\left(I_j\right), \overline{Q}_j\}$.

On the other hand, the best response of the k-th secondary user is $\mathbf{r}_{s,k}\left(\mathbf{a}_{-s,k}\right) = \mathbf{p}_k^* = (p_{k,1}^*, p_{k,2}^*, \cdots, p_{k,K_p}^*)^T$ where $\mathbf{p}_k^*$ is the *unique* transmitted power vector that maximizes $u_{s,k}$ subject to the power constraint $\|\mathbf{p}_k\| \leq \overline{P}_k$. Thus it can be shown that $\mathbf{p}_k^*$ should satisfies the following Karush-Kuhn-Tucker (KKT) conditions [9]: 1) $\mu_k \leq 0$, 2) $u_{k,j}'(p_{k,j}) + \mu_k = 0$ for all $j \in \mathcal{K}_p$, 3) $\mu_k \left(\sum_{j \in \mathcal{K}_p} p_{k,j} - \overline{P}_k\right) = 0$, and 4) $\sum_{j \in \mathcal{K}_p} p_{k,j} \leq \overline{P}_k$.

[1]The uniqueness in the best responses of the primary and secondary users is due to the concavity of the utility functions.



(a) The Game outcome and the secondary transmit power $p_{k,j}$.

(b) Primary user utility function $u_{p,j}$.

(c) The sum-rate and per-user rate of the secondary system.

Fig. 2. The D-DSL behavior at Nash equilibrium as a function of secondary system size K_s assuming identical secondary users , where $K_p = 4$, $\overline{Q}_j = 20$, $\overline{P}_k = 10$, $\rho^{(p)}_{0,k} = \rho^{(s)}_{k,0} = \rho^{(s)}_{k,k'} = 1$ and $h^{(j)}_{k,k'} = 1$.

V. Performance and Numerical Analysis of a D-DSL game based DSS system

In the following we investigate the performance at Nash equilibrium of the proposed D-DSL based DSS system. Unless stated otherwise, the system parameters are set as follows: the receiver noise variance is set to be $\sigma_{k,j} = 1$ for $j \in \mathcal{K}_p$ and $k \in \mathcal{K}_c$, the primary user target SINR is $\overline{\gamma}_{0,j} = 1$ for $\forall j \in \mathcal{K}_p$, the maximum possible primary interference caps are $\overline{Q}_j = 20$, the maximum secondary transmit power is $\overline{P}_k = 10$, the weighting coefficient is $\lambda_k = 1$ and $W_j = 1$. The cross correlation coefficients $\rho^{(p)}_{0,k} = \rho^{(s)}_{k,0} = \rho^{(s)}_{k,k'} = 1$ for all $k, k' \in \mathcal{K}_s$.

A. Identical secondary users: Ideal AWGN channel

It is interesting to first investigate the Nash equilibrium of a D-DSL system when all secondary users are identical, i.e. all channels are additive white Gaussian noise (AWGN): $h^{(j)}_{k,k'} = 1$ for all $k, k' \in \mathcal{K}_c$ and $j \in \mathcal{K}_p$. In Fig. 2(a), we have shown the interference cap Q^*_j, the actual secondary user interference I^*_j and the secondary user transmit powers $\|\mathbf{p}^*_k\|$ at the system Nash equilibrium. Note that due to the excess of resources in a D-DSL network, the total secondary transmit power in the multiple primary channel system is higher compared to that in the single primary channel case. Thus the interference caused by the secondary system in each channel is reduced whenever the number of primary channels is increased. Moreover, the behavior of interference cap Q^*_j follows I^*_j. For a small secondary system size, the secondary users fully utilize their resources by allocating all their available transmit powers among the primary channels while still causing a smaller interference level on each of the primary channels. Each primary user tries to compensate for the loss caused by the excess of resources by reducing the interference cap Q^*_j.

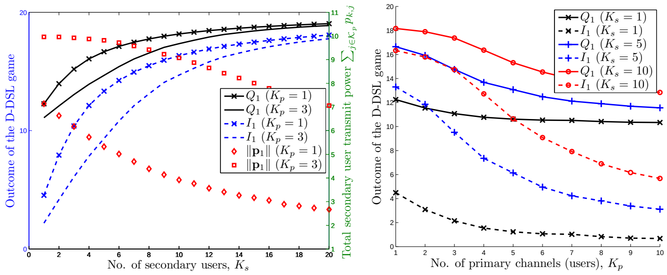
Figure 2(b) shows the primary utility at the Nash equilibrium of the system. When there is more resources (channels) available for the secondary system, the non-cooperative game among the primary users leads to decreased utility for all primary users. However, for large secondary system sizes, the demand for the spectrum resources is large enough so that the effect of the availability of multiple channels on primary utilities is again reduced.

Figure 2(c) shows the sum-rate $\sum_{k,j} f_{k,j}(p^*_{k,j})$ and the per-user rate $\frac{1}{K_s}\sum_{k,j} f_{k,j}(p^*_{k,j})$ as a function of the secondary system size. In the multiple primary channel scenario there is a gain in the rate for the secondary users when there is only one primary channel instead of four. This gain is expected since the secondary users have more choices when there are multiple primary channels. Thus they can transmit more power distributed over multiple channels compared to that in single channel.

B. Non-identical secondary users: Fading channels

In this section we assume Rayleigh distributed, quasi-static channel fading with normalized coefficients, i.e. $\mathbb{E}\left[\left(h^{(j)}_{k,k'}\right)^2\right] = 1$. In all simulations the results are averaged over 2000 channel realizations using Monte Carlo methods. Figure 3(a) shows the primary interference cap Q^*_j, the secondary interference I^*_j and the secondary transmit power $p^*_{k,j}$. Figure 3(b) shows how the interference cap Q^*_j as well as the total secondary interference I^*_j decrease with increasing number of primary channels K_p. This affects the primary user negatively. Due to competition among the primary users and the availability of multiple degrees of freedom for the secondary system, the reward of each primary user is reduced.

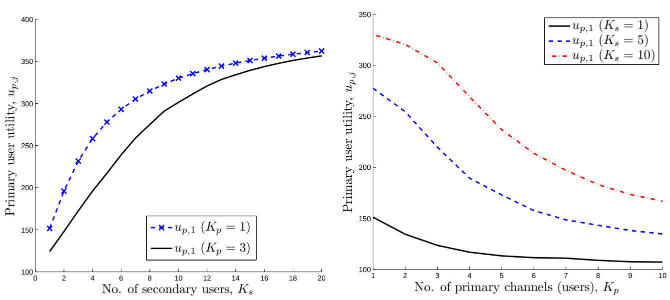
This reduction in primary utility with increasing K_p is seen in Fig. 4. As shown in Fig. 4(b), when the number of primary channels increases, the primary utility $u_{p,j}$ decreases. Figure 4(a) also shows that primary utility $u_{p,j}$ is still an increasing function of the secondary system size. Thus the primary users prefer having a large secondary system size (higher demand). However, as seen in Fig. 5(a), the secondary system has the incentive to keep K_s small enough to maintain a minimum QoS guarantee for all its users. Note that, due to the higher degrees of freedom available in a D-DSL system, the secondary system is able to achieve a better sum and per-user rates compared to those achieved in a single channel scenario. Due to the available degree of freedom, the secondary system can increase its total transmit power $\|\mathbf{p}_k\|$ without violating the primary user interference cap.



(a) The Game outcome and the secondary transmit power $p_{k,j}$ as a function of the secondary system size K_s.

(b) The Game outcome as a function of the primary system size K_p.

Fig. 3. The Game outcome and the secondary transmit power $p_{k,j}$, in a Rayleigh distributed channel fading.

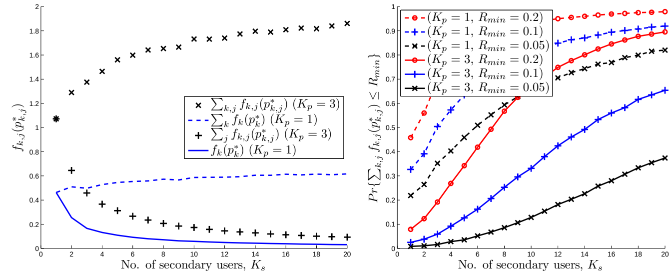


(a) Primary user utility function $u_{p,j}$ as a function of secondary system size K_s.

(b) Primary user utility function $u_{p,j}$ as a function of primary system size K_p.

Fig. 4. Primary user utility function $u_{p,j}$ at Nash equilibrium in a Rayleigh distributed channel fading.

Depending on the application, a secondary user may require a minimum rate to achieve a least acceptable QoS requirement. We denote this minimum rate for the secondary users by R_{min}. Because of the dependency on the random fading coefficients, at any given time a particular user may or may not achieve the rate found in Fig. 5(a). We define the outage probability as $Pr\{\sum_j f_{k,j}(p^*_{k,j}) \leq R_{min}\}$, which is the probability that a particular secondary user does not achieve the minimum required QoS. Figure 5(b) shows the outage probability of the secondary system. As one would expect the outage probability of the secondary users increases as the minimum QoS requirement R_{min} increases or as the number of primary channels decreases. Thus the average rate achieved by the secondary users shown in Fig. 5(a) should be interpreted in conjunction with the outage probability shown in Fig. 5(b). For instance, according to Fig. 5, with an average minimum rate of $R_{min} = 0.05$ the single channel D-DSL system can support up to 3 secondary users with an outage probability of 35% compared to the 3-channel D-DSL system which can handle up to 9 secondary users with a lower outage probability ($P_{out} = 0.1$).



(a) The sum-rate and per-user rate of the secondary system.

(b) The outage probability of the secondary system.

Fig. 5. The secondary system performance at Nash equilibrium as a function of secondary system size K_s.

VI. Conclusion

In this paper, we proposed a new architecture for DSS called the Distributed-DSL and developed a game-theoretic framework for implementing the proposed D-DSL in a cognitive radio network. In the proposed Distributed Dynamic Spectrum Leasing networks, the secondary users prefer a primary system size with a large number of distinct frequency channels. We showed that due to the multiple degrees of freedom available in a D-DSL system, the secondary system can achieve a better sum and per-user rates compared to those in a Centralized-DSL network. The outage probability is also improved with higher number of channels in the D-DSL system. We also showed that the increased number of degree of freedom negatively affects the primary users due to the competition caused by the non-cooperative game among the primary players.

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