# A Dynamic Spectrum Leasing (DSL) Framework for Spectrum Sharing in Cognitive Radio Networks

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Abstract-We propose dynamic spectrum leasing (DSL) as a new paradigm for dynamic spectrum sharing (DSS) in (cognitive) radio networks to improve spectrum utilization. In contrast to exiting hierarchical dynamic spectrum access (DSA), spectrum owners in proposed DSL networks are expected to dynamically adjust the amount of secondary interference they are willing to tolerate in response to the demand from secondary transmitters [1]. The secondary transmitters in turn attempt to achieve maximum possible throughput opportunistically while not violating the interference limit set by the primary system. In this paper, we first develop a formal signal model for DSL coexistence of primary and secondary systems, and then model their interactions as a non-cooperative DSL-game. We propose a class of utility functions for the two types of users based on demand and value, establish and analyze the equilibrium performance of a proposed DSL-based network and show how a practical implementation can be justified with minimal interaction between the two systems.

### I. INTRODUCTION

Several measurement campaigns in different countries have shown that the perceived scarcity of radio spectrum is mainly due to the inefficiency of traditional spectrum allocation policies [2], [3]. In some cases, it has been observed the allocated spectrum is not used by the incumbent owner for most of the time. This led the FCC to recommend three broad solutions to improve the spectrum utilization in its 2002 Spectrum Policy Task Force Report: a) spectrum reallocation, b) spectrum leasing, and c) spectrum sharing. Clearly, the first of these was meant to be a long-term solution. Perhaps the best example is the opening of the 700MHz TV band for cognitive radio operation. Spectrum leasing in [2] is also mostly interpreted to be a static or off-line solution, at least in current literature. The spectrum sharing solution, on the other hand, has spurred a flurry of research aimed at dynamic sharing of spectrum [4]–[7] (and references therein). The field is still seeing rapid research growth, and as a consequence there is much confusion in the terminology. In Fig. 1 we attempt to develop a taxonomy of various new schemes and concepts that have been identified for better spectrum utilization.

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Fig. 1. New solutions for better spectrum utilization.

Some of the spectrum sharing proposals can be identified as being hierarchical-access methods, in that there is usually a primary system that owns the spectrum rights and a secondary system that wants to access this spectrum whenever possible. However, this need not always be the case, as is true in Openaccess methods. In almost all existing hierarchical spectrum sharing proposals, however, the burden of interference management and coexistence is placed on the secondary system: the secondary system is expected to be vigilant of available spectrum opportunities and be able to efficiently use them. Thus, in Fig. 1 we term these proposals as the dynamic spectrum access (DSA). The DSA can be implemented in both spectrum underlay or spectrum overlay systems [8]. In DSA, the secondary users are expected to access the spectrum either only when primary users do not use their spectrum (overlay) or within a specific interference margin (underlay). This has led to cognitive radios as an enabling platform in realizing such dynamic spectrum sharing due to built-in cognition that can be used to observe, learn from and adjust to the RF interference [9]. Unfortunately though, the synonymous use of the term cognitive radio to imply dynamic spectrum sharing has added too much confusion and blurred definitions.

Recently, in [1] we proposed the new concept of *dynamic spectrum leasing* (DSL) as an approach for better spectrum utilization. The proposed DSL goes beyond (or re-interpret)

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the static spectrum leasing suggested under the second option above in which the spectrum licensees are granted the rights to sell or trade their spectrum to third parties [2], [8]. As opposed to passive spectrum sharing by the primary users as in existing DSA proposals, leasing means that the primary users have an explicit incentive (e.g. monetary rewards as leasing payments) to allow secondary users to access their licensed spectrum. In [1] we proposed to achieve this by allowing the primary users to dynamically adjust the extent to which they are willing to lease their spectrum. Unlike in DSA systems considered in existing literature, the primary users in a DSL network can actively manage the interference they see from the secondary transmissions by adapting their interference cap according to the observed RF environment and required Quality-of-Service (QoS). In this paper, we formalize the proposed DSL framework for dynamic spectrum sharing [1], [10]. Specifically, we first develop a signal and system model for DSL coexistence of primary and secondary systems (Section II). Next, in Section III we propose a game theoretic formulation to model the interactions among primary and secondary systems that captures the realities of such a DSL system. We develop a general structure for a suitable class of utility functions for both primary and secondary systems that reflect the demand for spectrum access from the secondary users, their payoffs in terms of a suitable performance measure and the primary user QoS requirements, and analyzes the conditions for reaching an equilibrium. In Section IV we provide several simulation results to demonstrate the properties of a DSL-based DSS system at the equilibrium, and use this to identify and resolve certain implementation issues. Finally, Section V concludes the paper by summarizing our ideas and findings.

## II. A SIGNAL AND SYSTEMS MODEL FOR DSL-BASED COEXISTENCE

We assume that there is one primary and one secondary wireless communication systems, each operating in the spectrum band of interest. For simplicity, it is further assumed that there is only a single transmitter-receiver pair in the primary system. There are K secondary *links* of interest. The primary user is denoted as user 0, and the secondary links are labeled as 1 through K. We will refer to k-th transmitter or k-th receiver to mean the transmitter and receiver of the k-th link. Depending on whether the secondary system is an infrastructurebased or an ad-hoc network, the receivers of each link may or may not be physically distinct. The channel gain between the k-th transmitter and the primary receiver and the j-th secondary receiver are denoted, respectively, by  $h_{pk}$  and  $h_{jk}$ for  $k = 0, 1, \dots, K$  and  $j = 1, \dots, K$ . We assume that M orthonormal directions specified by  $\{\psi_1^{(p)}(t), \dots, \psi_M^{(p)}(t)\}$ form a basis for the space-spanned by the transmit signals of the primary system. Assuming M discrete-time projections  $r_m^{(p)} = \langle r_p(t), \psi_m^{(p)}(t) \rangle$ , for  $m = 1, \cdots, M$ , of the continuous-time received signal  $r_p(t)$  on this *primary basis*, and letting  $\mathbf{r}^{(p)} = \left(r_1^{(p)}, \dots, r_M^{(p)}\right)^T$ , we obtain the discretetime representation of the received signal at the primary

receiver as  $\mathbf{r}^{(p)} = A_0 b_0 \mathbf{s}_0^{(p)} + \sum_{k=1}^K \Theta_k A_k b_k \mathbf{s}_k^{(p)} + \sigma_p \mathbf{n}^{(p)}$ , where assuming BPSK transmissions for all users  $b_k \in \{+1, -1\}$ ,  $A_k = \sqrt{p_k} h_{pk}$ ,  $p_k$  is the transmit power of the k-th user,  $\mathbf{s}_{k}^{(p)} = \left( \mathbf{s}_{k1}^{(p)}, \cdots, \mathbf{s}_{kM}^{(p)} \right)$  is the *M*-vector representation of the k-th user's transmit signal waveform  $s_k(t)$  w.r.t. the primary basis, where  $s_{km}^{(p)} = \langle s_k(t), \psi_m^{(p)}(t) \rangle$ , and  $\mathbf{n}^{(p)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_M)$ . Note that, for brevity we have assumed synchronous transmissions and dropped the symbol index in equation for  $\mathbf{r}^{(p)}$ . However, it should be noted that the model can indeed absorb the asynchronism with proper modifications. The Bernoulli random variable  $\Theta_k$  with  $Prob(\Theta_k = 1) = q_k$ and  $Prob(\Theta_k = 0) = 1 - q_k$ , represents the randomness of secondary-user collisions with the primary transmission. For example, in an overlay system  $q_k$  is the false-alarm probability of the white space detector of the k-th secondary link, while, in an underlay DSS system we may set  $q_k = 1$ .

Assuming matched filter (MF) receivers, primary decisions are given by  $\hat{b}_0 = sgn\left(y_0^{(p)}\right)$  where  $y_0^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{r}^{(p)} = A_0 b_0 + \sum_{k=1}^K \Theta_k \rho_{0k}^{(p)} A_k b_k + \sigma_p \eta^{(p)}$  with  $\rho_{0k}^{(p)} = \left(\mathbf{s}_0^{(p)}\right)^T \mathbf{s}_k^{(p)}$ . The total secondary interference  $I_0$  from all secondary transmissions to the primary-receiver is given by  $I_0 = \sum_{k=1}^K q_k^2 \left(\rho_{0k}^{(p)}\right)^2 A_k^2 = \sum_{k=1}^K \tilde{A}_k^2 p_k$  where  $\tilde{A}_k = q_k \rho_{0k}^{(p)} h_{p_k}$  is the effective channel coefficient of the k-th secondary user w.r.t. the primary receiver. This total interference parameter  $I_0$  plays a key role in any DSL-based DSS system, as we will see later.

Similarly, by assuming that the collection of waveforms  $\{\psi_1^{(s)}(t), \cdots, \psi_N^{(s)}(t)\}$  forms an N-dimensional orthonormal basis for the space spanned by the secondary transmit signals, a discrete-time representation of the received signal  $r_j^{(s)}(t)$  at the *j*-th secondary receiver, for  $j = 1, \dots, K$ , can be written as  $\mathbf{r}_j^{(s)} = \sum_{k=1}^{K_t} B_{j,k} b_k \mathbf{s}_k^{(s)} + B_{j,0} b_0 \mathbf{s}_0^{(s)} + \sigma_s \mathbf{n}_j^{(s)}$  where  $B_{j,k} = h_{jk} \sqrt{p_k}, \sigma_s^2$  is the variance of noise at the secondary receiver,  $n_{jk}\sqrt{p_k}, \sigma_s^{(s)}$  is the variance of noise at the secondary receiver,  $\mathbf{r}_j^{(s)} = \left(r_{j,1}^{(s)}, \cdots, r_{j,N}^{(s)}\right)^T$  with  $r_{j,n}^{(s)} = \langle r_j^{(s)}(t), \psi_n^{(s)}(t) \rangle$ , for  $n = 1, \cdots, N, \mathbf{s}_k^{(s)} = \left(\mathbf{s}_{k1}^{(s)}, \cdots, \mathbf{s}_{kN}^{(s)}\right)$  is the N-vector representation of  $s_k(t)$  w.r.t. the N-dimensional basis employed by the secondary system with  $s_{kn}^{(s)} = \langle s_k(t), \psi_n^{(s)}(t) \rangle$ , and  $\mathbf{n}_j^{(s)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}_N)$ . For simplicity, assume that the secondary detector to be based on the matched-filter, so that the j-th secondary receiver detects the corresponding j-th secondary transmitter's symbols as  $\hat{b}_j = sgn\left(y_i^{(s,j)}\right)$  where  $y_{i}^{(s,j)} =$  $\begin{pmatrix} \mathbf{s}_{k}^{(s)} \end{pmatrix}^{T} \mathbf{r}_{j}^{(s)} = B_{j,k} b_{k} + \sum_{k'=1,k'\neq k}^{K} \rho_{kk'}^{(s)} B_{j,k'} b_{k'} + \rho_{k0}^{(s)} B_{j,0} b_{0} + \sigma_{s} \eta_{k}^{(s,j)} \text{ with } \rho_{kk'}^{(s)} = \left( \mathbf{s}_{k}^{(s)} \right)^{T} \mathbf{s}_{k'}^{(s)}, \text{ for } k, k' = 0, 1, \cdots, K, \text{ and } \eta_{k}^{(s,j)} = \left( \mathbf{s}_{k}^{(s)} \right)^{T} \mathbf{n}_{j}^{(s)} \sim \mathcal{N}(0, 1).$  Spectrum sharing in proposed DSL networks is achieved by mutual interaction between primary and secondary systems. In particular, the primary users are to be proactive in managing the secondary access depending on the observed RF interference and QoS requirements. Note that, existing coexistence models invariably assume that only secondary users are the active participants in any such model while primary users are treated merely as passive spectators (see [4]–[6] and references therein). In PHY-layer DSLbased coexistence, we assume that the primary user action is

to dynamically set a maximum allowed secondary interference level  $Q_0$ . This *interference cap*  $Q_0$  in a DSL system indicates the maximum interference level the primary user is willing to tolerate from all secondary transmissions. At any given time, the primary user's target SINR is defined in terms of its assumed *worst-case* secondary interference:  $\bar{\gamma}_0 = \frac{h_{p0}^2 p_0}{Q_0 + \sigma_p^2}$ . On the other hand, the primary user's actual instantaneous SINR is given by  $\gamma_0 = h_{p0}^2 p_0 / \left( \sum_{k=1}^K q_k^2 \left( \rho_{pk}^{(p)} \right)^2 h_{pk}^2 p_k + \sigma_p^2 \right) =$  $\bar{\gamma_0}\left(1+\frac{Q_0-I_0}{I_0+\sigma_p^2}\right)$ . For notational simplicity, in the following we assume a common secondary receiver, at which the received SINR of the k-th secondary user is then given by  $\gamma_k = |h_{sk}|^2 p_k / (\sum_{j=1, j \neq k}^K (\rho_{kj}^{(s)})^2 |h_{sj}|^2 p_j + (\rho_{k0}^{(s)})^2 |h_{s0}|^2 p_0 + (\rho_{k0}^{(s)})^2$  $\sigma_s^2) = |h_{sk}|^2 p_k / (i_k + \tilde{\sigma}_s^2)$ , where  $\tilde{\sigma}_s^2 = \left(\rho_{k0}^{(s)}\right)^2 |h_{s0}|^2 p_0 + \sigma_s^2$ is the effective noise seen by the k-th user and  $i_k$  is the total secondary interference at the k-th secondary receiver.

#### **III. DSL GAMES**

We model the DSL as a noncooperative game  $(\mathcal{K}, \mathcal{A}_k, u_k(.))$ , where  $\mathcal{K} = \{0, 1, 2, \cdots, K\}$  is the player set of the DSL-game,  $\mathcal{P} = \mathcal{A}_0 \times \mathcal{A}_1 \times \mathcal{A}_2 \cdots \times \mathcal{A}_K$  is the action space with  $\mathcal{A}_0 = \mathcal{Q} = [0, \bar{Q}_0]$  representing the primary user's action set and  $A_k = P_k = [0, \bar{P}_k]$ , for  $k = 1, 2, \cdots, K$ , representing the k-th secondary link's (transmitters) action set. Note that  $Q_0$  and  $P_k$  represent, respectively, the maximum possible interference cap of the primary user and the maximum transmission power of the k-th secondary user (as determined by the system and regulatory considerations). We denote the action vector of all users by  $\mathbf{a} = [Q_0, p_1, \cdots, p_K]^T$ , where  $Q_0 \in \mathcal{Q}$  and  $p_k \in \mathcal{P}_k$ . It is customary to denote the action vector excluding the k-th user by  $\mathbf{a}_{-k}$ . Finally,  $u_0(Q_0, \mathbf{a}_{-0})$  is the primary user's utility function, and  $u_k(p_k, \mathbf{a}_{-k})$ , for  $k = 1, 2, \cdots, K$ , is the k-th secondary user's utility function. While exact choice may depend on the type of system in consideration, we propose a general class of utility functions for the primary user as

$$u_0(Q_0, \mathbf{a}_{-0}) = \left(\bar{Q}_0 - (Q_0 - I_0(\mathbf{a}_{-0}))\right) F(Q_0) = u_0(Q_0, I_0)$$
(1)

This  $u_0$  assumes that the utility of the primary user is proportional to both demand  $\bar{Q}_0 - (Q_0 - I_0)$  and a reward function F(.). Since demand decreases as extra interference margin  $Q_0 - I_0$  increases, it discourages the primary user to swamp all other transmissions (both primary and secondary) by setting too large an interference cap  $Q_0$  that will lead to higher transmission power. The reward function F is assumed to be a continuous and monotonic increasing function of its argument. The (selfish) objective of each secondary user is to maximize a given utility function that depends on its own SINR without violating the primary user interference cap. Observe from  $\gamma_0$  that as long as the secondary interference  $I_0$  is below the interference cap  $Q_0$  set by the primary user, the required primary QoS will be guaranteed. To ensure this, we propose the following form for the secondary user utility

function:

$$u_{k}(p_{k}, \mathbf{a}_{-k}) = (Q_{0} - \lambda_{s}I_{0}) f(p_{k})$$
  
=  $\left(Q_{0} - \lambda_{s}I_{0,-k} - \lambda_{s}\tilde{A}_{k}^{2}p_{k}\right) f_{k}(p_{k})$  (2)

where  $f_k(.)$  is a suitable, non-negative reward function,  $\lambda_s$  is a suitably chosen positive coefficient and  $I_{0,-k}$  =  $\sum_{j=1, j \neq k}^{K} \tilde{A}_{j}^{2} p_{j}$  is the total secondary interference to the primary user *excluding* that from the k-th secondary user. The coefficient  $\lambda_s$  in (2) controls how strictly secondary users need to adhere to the primary user's interference cap. The proposed utility function (2) leaves the performance metrics of the secondary system to be arbitrary by allowing for any reasonable reward function  $f_k(.)$  that will satisfy the conditions to be set forth below. Without loss of generality, we will assume that the reward function  $f_k(p_k)$  satisfies  $f_k(0) = 0$  and  $f'_k(0) > 0$ .

With above DSL-game formulation, the interesting question is how interactions among spectrum users will evolve when each user autonomously chooses an action that maximizes its self-interest, as quantified by utility. At a Nash equilibrium (NE) of the system no user has an incentive to unilaterally change its own strategy when all other users keep their strategies fixed. Hence, the NE is a stable outcome at which the DSL-game might reasonably end up. While we omit details here (see [10]), it can be shown that our DSL-game  $G = (\mathcal{K}, \mathcal{A}_k, u_k)$  with utility functions (1) and (2) indeed has a unique Nash equilibrium if the following conditions are satisfied (where we set  $f_k(p_k) = g_k(\gamma_k)$ ):

1. F and  $g_k$  are continuous and monotonic increasing functions of its arguments for  $k = 1, 2, \dots, K$ .

2. 
$$g(0) = 0, g'(0) > 0, \lim_{\gamma_k \to \infty} \frac{g(\gamma_k)}{g'(\gamma_k)} > -\infty$$
, and  $F(0) = 0, F'(0) > 0$  and  $\lim_{Q_0 \to \infty} \frac{F(Q_0)}{F'(Q_0)} > -\infty$ .

- 3.  $\frac{g(\gamma_k)g''(\gamma_k)}{(g'(\gamma_k))^2} < 2 \text{ for all } \gamma_k > 0, \text{ and } \frac{F(Q_0)F''(Q_0)}{(F'(Q_0))^2} < 2 \text{ for all } Q_0 > 0$ 4.  $0 < \lambda_s \leq \frac{Q_0}{I_{0,-k}}$

A best response correspondence of a user is the best reaction strategy  $r_k(.)$  a rational user k would choose in order to maximize its own utility, in response to actions chosen by other users. Due to uniqueness, if all users follow a bestresponse adaptation strategy, the system will indeed converge to the Nash equilibrium. A key advantage of the game theoretic formulation is that best-response adaptations lend themselves to distributed implementations.

It can be shown that the best response adaptations of the proposed DSL-game are  $r_0(\mathbf{a}_{-0}) = \min \{ \bar{Q}_0, Q_0^*(I_0) \}$  $\ge$  $r_0(I_0)$ , where  $Q_0^*(I_0)$  is the fixed-point solution to the equation  $x = \bar{Q}_0 + I_0 \frac{F(x)}{F'(x)}$ , and  $r_k(\mathbf{a}_{-k}) = \min\{\bar{P}_k, p_k^*(Q_0, I_{0,-k}, i_k)\}$ , for  $k = 1, \cdots, K$ , where  $p_k =$  $p_{k}^{*}(Q_{0}, I_{0,-k}, i_{k})$  is the unique solution to equation  $\phi_{k}(\gamma_{k})$  –  $\frac{1}{\lambda_s} = 0, \text{ with } \phi_k(\gamma_k) = \frac{I_{0,-k}}{Q_0} + \frac{\tilde{A}_k^2 N_k}{Q_0} \left(\gamma_k + \frac{g(\gamma_k)}{g'(\gamma_k)}\right).$ These best responses shed lights on what information each

user needs to know about the others, and in particular, between the two systems. The proposed model has been carefully constructed in order to capture the essential inter-dependence between the two systems while ensuring that the best response

adaptations can be achieved with minimal conscious intersystem information exchanges. Indeed, the only quantity the primary user needs to determine its best response for a chosen action vector  $\mathbf{a}_{-0}$  by the secondary users, is the total secondary interference  $I_0$  at the primary receiver, which can be easily be estimated at the primary receiver. Similarly, the best response of the k-th secondary user is a function of  $Q_0$ , the residual interference  $I_{0,-k}$  from all other secondary users to the primary user, and the total interference from all users to the k-th user's received signal at the secondary receiver. The secondary system can estimate the latter quantity. In a DSL-based network, it is assumed that the primary system periodically broadcasts  $Q_0$  and  $I_0$ , providing  $Q_0$  and  $I_{0,-k}$ . Since  $Q_0$  and  $I_0$  are readily available at the primary system, the periodic broadcast of these quantities is a reasonable expectation for future DSL-based networks. Note that in the existing literature it is always assumed that the primary system already broadcasts the so-called maximum tolerable *interference temperature*, which is equivalent to  $Q_0$ . Observe that, knowing  $I_0$  each secondary user can compute the residual interference  $I_{0,-k} = I_0 - A_k^2 p_k$  since it knows its own transmit power and it may estimate the channel state information  $A_k$ if the reverse link signals are available in the same band. Otherwise, we propose the approximation  $I_{0,-k} \approx I_0$  to be used. As we will show in the next section, this performs well in practice.

#### IV. EQUILIBRIUM COEXISTENCE OF DSL SYSTEMS

In the following we investigate the primary and secondary system DSL-coexistence within each other's required performance QoS constraints and based on that identify certain design guidelines. To be specific, we choose the primary secondary reward functions to be  $F(Q_0) = Q_0$  and  $f_k(p_k) =$  $g(\gamma_k) = W \log(1 + \gamma_k)$ , for  $k = 1, \dots, K$ . It can readily be verified that these satisfies the above conditions for the existence of an NE. Note that, these rewards can easily be motivated for a DSL system, and we omit details here [10].



Fig. 2. Best response functions and the secondary utility in a K = 2 (identical users) dynamic spectrum leasing network when  $\lambda_s = 1$ ,  $\bar{Q} = 10$ ,  $\bar{P}_k = 12$ ,  $W_k = 1$ ,  $R_k = 1$ ,  $\bar{\gamma}_0 = 1$ ,  $q_k = 1$ ,  $h_{p0} = h_{sk} = 1$ ,  $\sigma_s^2 = \sigma_p^2 = 1$ ,  $\rho_{0k}^{(p)} = \rho_{kj}^{(s)} = 1$ ,  $h_{pk} = h_{sk} = 1$  for all k, and  $\sigma_s^2 = \sigma_p^2 = 1$ . (a) Best-response functions. (b) First secondary user utility  $u_1(p_1)$  when all other secondary users and the primary user keep their actions fixed at Nash equilibrium profile, i.e.  $Q_0 = Q_0^* = 7.39$  and  $p_2 = p_2^* = 2.39$ .

Figure 2(a) shows the best-response adaptations in secondary system comprised of K = 2 identical users. In this case, we assume that  $\rho_{0k}^{(p)} = \rho_0^{(p)}$ ,  $\rho_{k0}^{(s)} = \rho_0^{(s)}$ ,  $\rho_{kj}^{(s)} = \rho^{(s)}$ , for all  $k, j = 1, \dots, K$ , same collision probabilities  $q_k = q$ , for all k and all channels are additive white Gaussian noise (AWGN):  $h_{sk} = h_{pk} = 1$  for all  $k = 0, 1, \dots, K$ . Then  $\tilde{A}_k = \tilde{A}$  for all k. By symmetry, in this case all secondary users must have the same power  $p_k = p^*$  at the Nash equilibrium (equivalently, the same SINR  $\gamma_k = \gamma^*$ ), and indeed we can analytically solve this Nash profile [10]. Here, from the figure we observe that the unique Nash equilibrium action profile in this system is specified by  $(Q_0^*, p^*) = (7.39, 2.39)$ . Figure 2(b) shows the first secondary user utility as a function of its transmit power when the primary user and the other secondary user keep their actions fixed at the Nash profile.



Fig. 3. Outcome of the DSL game at the system Nash equilibrium, with both exact CSI and using the approximation  $I_{0,-k} \approx I_0$ , as a function of secondary system size K in the presence of channel fading when  $\lambda_s = 1$ . All channel fading coefficients are Rayleigh with  $\mathbb{E}\{h^2\} = 1$ .  $\bar{Q} = 10$ ,  $\bar{P}_k = 10$ ,  $W_k = 1$ ,  $R_k = 1$ ,  $\bar{\gamma}_0 = 1$ ,  $q_k = 1$ ,  $\rho_{(k)}^{(p)} = \rho_{(k)}^{(s)} = 1$ , and  $\sigma_s^2 = \sigma_p^2 = 1$ .

In the presence of wireless channel fading, the Nash equilibrium power profile of the dynamic spectrum leasing system will depend on the observed channel state realization. In particular, the Nash equilibrium transmit powers of individual secondary users will be different for each user. In Fig. 3 we have shown the game outcome at the Nash equilibrium in the presence of channel fading as a function of the number of secondary users K, both with and without CSI. We assume all channel gains in the system to follow a Rayleigh distribution with all coefficients normalized so that  $\mathbb{E}\{h^2\} = 1$ .

From Fig. 3 we can observe how the total interference  $I_0$ increases with increasing K, and how, in turn, the primary user also increases its interference cap to maximize its utility. It is also of interest to note that the safety margin  $Q_0 - I_0$ is large for smaller number of users, and seems to decrease with increasing K. This is essentially due to the fact that the number of degrees of freedom in a multiuser system is being proportional to the number of users. When K is large, the interference generated by the secondary system  $I_0$  is close to the interference cap  $Q_0$ , yet, as desired, is always below it. Note that, when exact CSI  $I_{0,-k}$  is not available, the secondary users in Fig. 3 employ the blind approximation  $I_{0,-k} \approx I_0$ . As we may observe from Fig. 3, still the DSL game converges to a Nash equilibrium that does not violate the primary interference cap. The only effect of not having the exact  $I_{0,-k}$  is that the safety margin  $Q_0 - I_0$  at the equilibrium is slightly larger. This is because each secondary user believes an exaggerated



Fig. 4. Achieved utilities and secondary outage probabilities at the system NE in fading channels  $f_k(p_k) = g(\gamma_k) = \log(1 + \gamma_k)$  and  $\lambda_s = 1$ . All fading coefficients are Rayleigh with  $\mathbb{E}\{h^2\} = 1$ .

residual interference  $I_{0,-k}$ . Figures 4(a) and 4(b) show the corresponding primary and secondary user utilities achieved at the NE in the presence of channel fading. In Fig. 4 we have set  $W_k = W = 1$  so that the secondary reward with  $f = f^{(1)}$  has the meaning of spectral efficiency in bits-persecond-per-Hertz. The reward for a secondary user in this case is the capacity (in bps) it can achieve assuming all other transmissions (both primary and secondary) are purely noise. In the presence of channel fading, this capacity is a random quantity determined by the fading coefficients of all users. As is observed from Fig. 4(b), the per-user reward is typically decreasing in the increasing secondary system size. The interpretation is simple: All secondary users in the system must share the allowed interference level set by the primary system. However, a secondary user may require a minimum capacity (or a rate) to ensure an acceptable QoS for its applications. We define this minimum transmission quality as the average (over fading) minimum reward achieved by a user at the equilibrium, and denote by  $f_{\min,k}$ , for user k. In all simulation results below we assume that  $f_{\min,k} = f_{\min}$ for all secondary users. While we omit details here due to space, it can be observed from simulated results that as the minimum QoS requirement  $f_{\min}$  increases, the number of secondary users who can simultaneously transmit decreases. In addition, the maximum secondary system size also decreases, albeit slowly, as the pricing coefficient  $\lambda_s$  increases. We may also observe that the greatest impact of the coefficient  $\lambda_s$  is on the primary system, and when  $\lambda_s < 1$  there is a high likelihood that the interference cap might be exceeded by even a relatively smaller size secondary systems. Thus in a DSL network, the primary system must set the pricing coefficient  $\lambda_s$  based on how strictly it wants the secondary users to adhere to the maximum interference cap condition.

At times, depending on the fading statistics a particular user may or may not meet the minimum transmission quality at the system equilibrium. When this occurs we say that the user is in outage and thus the probability of outage for user k is defined as  $Pr(f_k(p_k^*) < f_{\min,k})$ . Figure 4(c) shows the outage probability of a typical secondary user. It is seen that the outage probability increases with K as well as with the minimum QoS requirement. The maximum secondary system size which can be supported thus needs to be interpreted in conjunction with the outage probabilities shown in Fig. 4(c).

## V. CONCLUSION

We proposed a new framework for dynamic spectrum sharing, called dynamic spectrum leasing, or DSL for short, that has the potential to further improve the efficient coexistence of primary and secondary wireless systems by requiring the primary systems to be explicitly proactive. In this paper we formulated the PHY layer interactions in a DSL network as a non-cooperative game and developed a suitable DSLgame that captures the essential element. The equilibrium performance of a DSL game was analyzed and the required control information exchange was identified. In particular, it was shown that indeed DSL can be implemented with minimal information exchange between the primary and secondary systems similar to that required in existing DSA proposals.

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