# Analysis of Linear Receivers in a Target SINR Game for Wireless Cognitive Networks

Tianming Li

Department of Electrical and Computer Engineering University of New Mexico Albuquerque, NM 87131 USA Email: kevinltm@ece.unm.edu

Abstract-Signal to Interference plus Noise Ratio (SINR) is a key parameter for every user in a wireless network. Different users with heterogeneous QoS requirements have different target SINR requirements. In cognitive radio (CR) networks, secondary users try to access the available spectrum in order to make successful transmissions. However, without proper regulation, they may transmit at their maximum power to achieve the highest possible SINR, which can be even worse than the current wasteful static spectrum utilization. A target SINR game (TSG) is a powerful tool to regulate each secondary user's behavior, provide them with decent SINRs (i.e. close to their target SINRs) and simultaneously limit the interference they cause to primary users and other secondary users. The goal of this paper is to analyze the performance of the Matched Filter (MF) receiver and the linear MMSE receiver (LMMSE) in a TSG. As expected, the LMMSE shows several advantages in performance over the MF.

*Index Terms*—Cognitive radios, target SINR game, game theory, Nash equilibrium, LMMSE receiver.

## I. INTRODUCTION

Game theory is a collection of tools for analyzing the interaction among rational decision makers. In a wireless network, different users are the players who compete for the valuable resource: spectrum. Many researchers have used the game theoretical methods to analyze the resource allocation in wireless networks. In [1], the authors proposed an energy efficient utility function that was shown to have a unique Nash Equilibrium (NE) due to its quasi-concavity property. In [2], by realizing the NE in the game in [1] may not be optimum, the authors further introduced the concept of Pareto efficiency into the game. They imposed a linear pricing function to gain better overall performance. [3] generalized this energy efficiency game by using Linear Minimum Mean Squared Error receiver (LMMSE) and showed that the game also converges to a unique NE due to the quasi-concavity property of the utility function. In [4], the authors generalized this game further by considering the QoS constraints. A summary on game theoretical approaches used in the energy efficient resource allocation in wireless networks can be found in [5].

One of the key objectives of cognitive radio (CR) networks is to allow unlicensed secondary users to share the available spectrum that is assigned to the primary users. This can greatly increase the spectrum efficiency. However, if all secondary users pour large amounts of power into the Sudharman K. Jayaweera

Department of Electrical and Computer Engineering University of New Mexico Albuquerque, NM 87131 USA Email: jayaweera@ece.unm.edu

available spectrum, the primary users' QoS can be greatly undermined. Hence, power control is of paramount importance in a cognitive network. An adherence to hierarchies between primary and secondary users in a peer-to-peer CR network through distributed power control was introduced in [6]. A more efficient branch and bound algorithm was proposed in [7] for optimal power control in cognitive radio network and an adaptive transmission scheme based on the Interference Temperature was analyzed in [8]. As another solution, [9] introduced the TSG model which can provide each secondary user with an acceptable Signal to Interference plus Noise Ratio (SINR) while limiting the secondary users' transmission power simultaneously. In this paper, their results on the matched filter receiver are re-derived in a new method and also the TSG is generalized to linear-minimum-mean-squared error receiver (LMMSE).

The remainder of this paper is organized as follows: Section II introduces the system and the TSG game models. Sections III and IV show that TSG with both the MF receiver and the LMMSE receiver have unique Nash Equilibria that can be reached by the best response algorithm. In section V, the numerical results are used to show that the TSG with the LMMSE receiver has advantages that are important in practice over that of the MF receiver. Section VI concludes this paper.

#### II. SYSTEM AND TSG GAME MODELS

#### A. System Model

Suppose that we have a single cell DS-CDMA wireless system with uniformly distributed K secondary cognitive radios and a single receiver [9]. All cognitive radios compute their instant SINRs from the broadcast information of the common receiver. Based on these SINRs, the cognitive radios adapt their transmit power in order to bring their SINRs closer to their target SINRs. The secondary radios' SINRs are determined by their heterogeneous QoS requirements

The received signal at the common receiver can be written as:

$$r(t) = \sum_{k=1}^{K} \sum_{i=0}^{B-1} A_k b_k s_k (t - iT) + \sigma n(t)$$

where  $n(t) \sim \mathcal{N}(0, 1)$  is the standard additive white Gaussian noise,  $A_k$  is the k-th user received signal amplitude,  $b_k(i)$  is



Fig. 1. Network Topology

the *i*-th symbol of the *k*-th user and  $s_k(t)$  is the *k*-th user's spreading waveform with  $\int_{-\infty}^{\infty} s_k^2(t) dt = 1$  and  $s_k(t) \neq 0$  only if  $t \in [0, T]$ , where T is the symbol duration.

The conventional detector for the k-th user is:

$$\hat{b}_k(i) = \operatorname{sgn}(y_k(i))$$
 for  $i = 0, 1, ..., B - 1$ ,

where

$$y_k(i) = \int_{-\infty}^{\infty} r(t)s_k(t - iT)dt,$$
  
=  $A_k b_k(i) + \sum_{j=1, j \neq k}^{K} A_j b_j(i)\rho_{j,k} + n_k(i)$ 

with

$$\rho_{j,k} = \int_{-\infty}^{\infty} s_j(t) s_k(t) dt = \mathbf{s}_j^T \mathbf{s}_k, \tag{1}$$

and

$$n_k(t) = \int_{-\infty}^{\infty} \sigma n(t) s_k(t - iT) dt \sim N(0, \sigma^2).$$

Note that in (1),  $s_k$  denotes the spreading code of user k. Thus, the k-th user's SINR is

$$\gamma_k = \frac{A_k^2}{\sum_{j=1, j \neq k}^K A_j^2 \rho_{j,k}^2 + \sigma^2},$$

#### B. Game Model

In this system, if all users want to achieve an SINR as high as possible, they may transmit at their maximum power introducing significant interference to the system. Thus, according to different QoS requirements of users, different target SINRs should be assigned to users. Every user should try to achieve its target SINR while limiting its transmit power in order to reduce the interference it causes to others. Here, we use a game theoretical approach to analyze power allocation in this system. We introduce following definitions:

- 1) Player set K: Secondary cognitive radios  $k \in K$ .
- 2) Action space  $\bar{A}$ :  $\bar{A} = \bar{A}_1 \times \bar{A}_2 \times ... \times \bar{A}_K$ , where  $\bar{A}_k = [0, A_{MAX}]$  is the k-th radio's received signal amplitude

set and  $A_{MAX}$  is the maximum received signal power. We let  $A_k = \sqrt{p_k \times g_k}$ ,  $A_k \in \overline{A}_k$ , where  $p_k$  is the k-th user's transmit power and  $g_k$  is its channel gain. While in practice the users adapt their transmit power to achieve the target SINR, for theoretical analysis, we assume that the channel is static in this game, i.e.  $g_k$  is constant, for  $\forall k \in K$ . Then,  $p_k = \frac{A_k^2}{g_k}$ , where the adaptation of the received signal amplitude  $A_k$  can reveal that of the transmit power.

3) Utility function  $u_k, \forall k \in K$ :

$$u_k(\mathbf{A}) = -\left(\gamma_T(k) - \gamma_i(k)\right)^2 \tag{2}$$

Here,  $\gamma_T(k)$  and  $\gamma_i(k)$  are the target and instantaneous SINRs of user k.  $\mathbf{A} = (A_1, A_2, ..., A_K)$  is the received signal amplitude vector, which is referred as the network state.

In this game model, all secondary radios adapt their transmit powers to satisfy their target SINRs that represent their QoS requirements. Note that, game (2) limits the interference a radio causes to the system and simultaneously protects the radio's benefit. At an NE of this game, if one exists, no secondary radio will have an incentive to change its transmit power.

## III. THE ANALYSIS OF TSG WITH THE MATCHED FILTER Receiver

If we are to use a matched filter as the common receiver, the utility function of the k-th radio becomes

$$u_{k}(\mathbf{A}) = -(\gamma_{T}(k) - \gamma_{i}^{MF}(k))^{2}$$
  
=  $-(\gamma_{T}(k) - A_{k}^{2}I_{k}^{-1})^{2}$   
=  $-(I_{k}^{-1})^{2}A_{k}^{4} + 2\gamma_{T}(k)I_{k}^{-1}A_{k}^{2} - \gamma_{T}^{2}(k),$  (3)

where  $I_k = \sum_{j=1, j \neq k}^{K} A_j^2 \rho_{j,k}^2 + \sigma^2$  is the interference the k-th user suffers from other secondary users.

## A. Existence of a Nash Equilibrium

From Theorem 11 in [2], a Nash equilibrium exists in game  $G = (K, \overline{A}, u_k(.))$ , if for all k = 1, 2, ..., K:

- 1) The k-th user's action set,  $\bar{A}_k$ , is a nonempty convex, and compact subset of some Euclidean space  $\mathbb{R}^N$
- 2)  $u_k(\mathbf{A})$  is continuous in  $\mathbf{A}$  and quasi-concave in  $A_k$ .

Obviously, since the k-th user's action set  $\bar{A}_k$  is a closed and bounded interval, it satisfies the first condition. Furthermore,  $u_k(\mathbf{A})$  in (3) is continuous in  $\mathbf{A}$  and concave in  $A_k$ . Thus, at least one Nash equilibrium exists in the TSG with the MF receiver.

#### B. Uniqueness of the Nash Equilibrium

**Definition 1** A best response correspondence,  $\forall k \in K$ ,  $r_k : \bar{A}_{-k} \to \bar{A}_k$ , is

$$r_k(\mathbf{A}_{-k}) = \{A_k \in \bar{A}_k : u_k(A_k, \mathbf{A}_{-k}) \ge u_k(A'_k, \mathbf{A}_{-k}) \ \forall A'_k \in \bar{A}_k\}$$
(4)

**Definition 2** Round robin best response decision rule.

In every adaptation round, every player updates its action sequentially. In particular, the k-th player  $\forall k \in K$ , updates its action to its best response action.

To establish the uniqueness of the NE in TSG for the MF receiver, we first show that the best response correspondence  $\mathbf{r}(\mathbf{A})$ , i.e.  $\mathbf{r}(\mathbf{A}) = (r_1(\mathbf{A}), r_2(\mathbf{A}), \dots, r_K(\mathbf{A}))$  is a standard function. From [10],  $\mathbf{r}(\mathbf{A})$  is a standard function if

1) positivity:  $\mathbf{r}(\mathbf{A}) > 0$ 

- 2) monotonicity: if  $\mathbf{A} \ge \mathbf{A}'$ , then  $\mathbf{r}(\mathbf{A}) \ge \mathbf{r}(\mathbf{A}')$
- 3) scalability: for all  $\mu > 1$ ,  $\mu \mathbf{r}(\mathbf{A}) > \mathbf{r}(\mu \mathbf{A})$ .

All the above inequalities are component-wise, i.e.  $r(A) \ge c$  $\mathbf{r}(\mathbf{A}') \Leftrightarrow r_k(\mathbf{A}) \ge r_k(\mathbf{A}') \ \forall k \in K.$ 

The best response correspondence for game (3) can be solved by setting

$$\frac{\partial u_k(\mathbf{A})}{\partial A_k} = 4A_k I_k^{-1} (\gamma_T(k) - \gamma_i^{MF}(k)) = 0, \qquad (5)$$

so that, the best response correspondence of the k-th user is

$$A_k^* = r_k(\mathbf{A}) = \sqrt{\frac{\gamma_T(k)}{I_k^{-1}}} \quad \forall k \in K.$$
(6)

In the following, we show that the best response correspondence (6) satisfies the above three conditions.

- 1) positivity: From (6), since  $\gamma_T(k) > 0$ , the best response correspondence satisfies the positivity condition above.
- 2) monotonicity: Given  $\mathbf{A} \geq \mathbf{A}'$ , i.e.  $A_k \geq A'_k$ ,  $\forall k = 1, 2, ..., K$ . Since  $I_k = \sum_{j=1, j \neq k}^{K} A_j^2 \rho_{j,k}^2 + \sigma^2$ , and  $I'_k = \sum_{j=1, j \neq k}^{K} A'_j^2 \rho_{j,k}^2 + \sigma^2$ , then we have that  $I_k^{-1} \leq I'_k^{-1}$ . Hence

$$r_k(\mathbf{A}) = \sqrt{\frac{\gamma_T(k)}{I_k^{-1}}}$$
$$\geq \sqrt{\frac{\gamma_T(k)}{I_k^{-1}}}$$
$$= r_k(\mathbf{A}')$$

3) scalability: For  $\forall \mu > 1$  and  $\forall k \in K$ 

$$\mu r_k(\mathbf{A}) = \mu \sqrt{\frac{\gamma_T(k)}{\left(\sum_{j=1, j \neq k}^K A_j^2 \rho_{j,k}^2 + \sigma^2\right)^{-1}}}$$
$$= \sqrt{\frac{\gamma_T(k)}{\left(\sum_{j=1, j \neq k}^K \mu^2 A_j^2 \rho_{j,k}^2 + \mu^2 \sigma^2\right)^{-1}}}$$
$$> \sqrt{\frac{\gamma_T(k)}{\left(\sum_{j=1, j \neq k}^K \mu A_j^2 \rho_{j,k}^2 + \sigma^2\right)^{-1}}}$$
$$= r_k(\mu \mathbf{A})$$

Thus,  $\mu \mathbf{r}(\mathbf{A}) > \mathbf{r}(\mu \mathbf{A})$ .

Thus, the best response correspondence of TSG with the MF receiver is a standard function. It has been shown that the fixed point  $\mathbf{A} = \mathbf{r}(\mathbf{A})$ , i.e. a Nash Equilibrium, is unique when  $\mathbf{r}(\mathbf{A})$  is a standard function [10]. Thus, the Nash equilibrium in this game is unique.

When the game converges, the action vector arrives at a steady state, meaning that given other players' actions are fixed, the k-th user doesn't have any incentive to change its action, i.e a Nash Equilibrium of this game. We can conclude that under the best response algorithm, all the steady states are NEs. In conclusion, the TSG with the MF receiver has a unique NE and converges to this NE under round robin best response decision rule.

## IV. THE ANALYSIS OF THE TSG WITH THE LINER MMSE RECEIVER

It is well known that linear MMSE receiver maximizes the output SINR. Thus, with the same target SINR constraints, the linear MMSE receiver may require the secondary radios to transmit at a lower power than that with the matched filter receiver. When the common receiver utilizes the linear MMSE receiver, the SINR for the k-th secondary radio can be characterized as [11]:

$$\gamma_i^{MMSE}(k) = A_k^2 \mathbf{s}_k^T \boldsymbol{\Sigma}_k^{-1} \mathbf{s}_k = \frac{A_k^2}{I_k}$$

where  $I_k = \left(\mathbf{s}_k^T \boldsymbol{\Sigma}_k^{-1} \mathbf{s}_k\right)^{-1}$  is the interference the k-th user suffers from other secondary users and  $\Sigma_{\mathbf{k}} = \sum_{j \neq k} A_j^2 \mathbf{s}_j \mathbf{s}_j^T +$  $\sigma^2 I$ . Then, the TSG for the LMMSE receiver is the same as (3).

In the following, we show that the TSG with the LMMSE receiver also has a unique NE.

### A. Existence of A Nash Equilibrium

Similar to the discussion in the section III-A,  $u_k(\mathbf{A})$  is continuous in A and quasi-concave in  $A_k$ . Also, since the k-th user's action set  $\bar{A}_k$  is a closed and bounded interval, it is a nonempty, convex and compact subset of  $\mathbb{R}^1$ . Thus, at least one Nash equilibrium exists in the TSG with the LMMSE receiver.

#### B. Uniqueness of the Nash Equilibrium

To establish the uniqueness of the NE, we show that the best response correspondence of the TSG with the LMMSE is also a standard function.

**Proposition 1** If two  $n \times n$  matrices A and B are both real, symmetric and positive definite, such that  $\mathbf{B} - \mathbf{A} > \mathbf{0}$ (i.e.  $\mathbf{B} - \mathbf{A}$  is positive semi-definite), then  $\mathbf{A}^{-1} - \mathbf{B}^{-1} \ge \mathbf{0}$ . In particular, when  $\mathbf{B} - \mathbf{A} > \mathbf{0}$ , than  $\mathbf{A}^{-1} - \mathbf{B}^{-1} > \mathbf{0}$ . Proof:

$$\mathbf{B} - \mathbf{A} \ge \mathbf{0} \Rightarrow \mathbf{I} - \mathbf{B}^{-\frac{1}{2}} \mathbf{A} \mathbf{B}^{-\frac{1}{2}} \ge \mathbf{0}.$$

Then,

$$\begin{aligned} 0 &< \min\left\{ \operatorname{eig}(\mathbf{B}^{-\frac{1}{2}}\mathbf{A}\mathbf{B}^{-\frac{1}{2}}) \right\} \leq \max\left\{ \operatorname{eig}(\mathbf{B}^{-\frac{1}{2}}\mathbf{A}\mathbf{B}^{-\frac{1}{2}}) \right\} \leq 1 \\ \Leftrightarrow 1 &\leq \min\left\{ \operatorname{eig}\left( (\mathbf{B}^{-\frac{1}{2}}\mathbf{A}\mathbf{B}^{-\frac{1}{2}})^{-1} \right) \right\} \leq \max\left\{ \operatorname{eig}\left( (\mathbf{B}^{-\frac{1}{2}}\mathbf{A}\mathbf{B}^{-\frac{1}{2}})^{-1} \right) \right\} < +\infty \\ \Leftrightarrow (\mathbf{B}^{-\frac{1}{2}}\mathbf{A}\mathbf{B}^{-\frac{1}{2}})^{-1} - \mathbf{I} > \mathbf{0}. \end{aligned}$$

Hence we have

$$\mathbf{B}^{\frac{1}{2}}\mathbf{A}^{-1}\mathbf{B}^{\frac{1}{2}} - \mathbf{I} \ge \mathbf{0}$$
$$\Leftrightarrow \mathbf{A}^{-1} - \mathbf{B}^{-1} \ge \mathbf{0}.$$

Similarly, when  $\mathbf{B} - \mathbf{A} > \mathbf{0}$ , the strict inequality can be established.

From (6), the best response correspondence is:

$$A_k^* = r_k(\mathbf{A}) = \sqrt{\frac{\gamma_T(k)}{I_k^{-1}}}, \quad \forall k \in K$$

where  $I_k = \left(\mathbf{s}_k^T \boldsymbol{\Sigma}_k^{-1} \mathbf{s}_k\right)^{-1}$ .

Note that, if we let  $\Sigma_{\mathbf{k}} = \sum_{j \neq k} A_j^2 \mathbf{s}_j \mathbf{s}_j^T + \sigma^2 I$ , then clearly  $\Sigma_{\mathbf{k}}$  is symmetric. In the following, we show that the best response correspondence of the TSG with the LMMSE receiver is also a standard function.

- 1) positivity: From (6), since  $\gamma_T(k) > 0$ , the best response correspondence satisfies the positivity condition.
- 2) monotonicity: For user  $k, \forall k \in K$

$$r_k(\mathbf{A}) = \sqrt{\frac{\gamma_T(k)}{I_k(\mathbf{A})^{-1}}},$$

and

$$r_k(\mathbf{A}') = \sqrt{rac{\gamma_T(k)}{I_k(\mathbf{A}')^{-1}}}$$

where  $I_k(\mathbf{A}) = (\mathbf{s}_k^T \boldsymbol{\Sigma}_k(\mathbf{A})^{-1} \mathbf{s}_k)^{-1}$ . If  $\mathbf{A} \ge \mathbf{A}'$ , then  $A_k \ge A'_k$ ,  $\forall k = 1, 2, ..., K$ . Therefore,

$$\boldsymbol{\Sigma}_{\mathbf{k}}(\mathbf{A}) - \boldsymbol{\Sigma}_{\mathbf{k}}(\mathbf{A}') = \sum_{j \neq k} \alpha_j \mathbf{s}_j \mathbf{s}_j^T,$$

where  $\alpha_j = A_j^2 - (A_j')^2 \ge 0$ . Hence,  $\sum_{j \neq k} \alpha_j \mathbf{s}_j \mathbf{s}_j^T$  is positive semi-definite and symmetric. Thus,  $\boldsymbol{\Sigma}_k(\mathbf{A}) - \boldsymbol{\Sigma}_k(\mathbf{A}') \ge \mathbf{0}$ .

Since  $\Sigma_{\mathbf{k}}(\mathbf{A})$  and  $\Sigma_{\mathbf{k}}(\mathbf{A}')$  are both real, symmetric and positive definite matrices, from Proposition 1, we have that

$$\begin{split} & \boldsymbol{\Sigma}_{\mathbf{k}}(\mathbf{A}')^{-1} - \boldsymbol{\Sigma}_{\mathbf{k}}(\mathbf{A})^{-1} \geq \mathbf{0} \\ & \Leftrightarrow \mathbf{s}_{k}^{T} \boldsymbol{\Sigma}_{\mathbf{k}}(\mathbf{A}')^{-1} \mathbf{s}_{k} - \mathbf{s}_{k}^{T} \boldsymbol{\Sigma}_{\mathbf{k}}(\mathbf{A})^{-1} \mathbf{s}_{k} \geq 0 \\ & \Leftrightarrow I_{k}(\mathbf{A}')^{-1} - I_{k}(\mathbf{A})^{-1} \geq 0. \end{split}$$

Hence, given the same  $\gamma_T(k)$ ,

$$r_k(\mathbf{A}) \ge r_k(\mathbf{A}').$$

3) scalability: For  $\mu > 1$  and  $\forall k \in K$ , let

$$\mathbf{Q_1} = \sum_{j \neq k} \mu^2 A_j^2 \mathbf{s}_j \mathbf{s}_j^T + \sigma^2 \mathbf{I}_j$$

and

$$\mathbf{Q_2} = \sum_{j \neq k} \mu^2 A_j^2 \mathbf{s}_j \mathbf{s}_j^T + \mu^2 \sigma^2 \mathbf{I}.$$

Then,

$$\begin{aligned} r_k(\mu \mathbf{A}) &= \sqrt{\frac{\gamma_T(k)}{\left(\mathbf{s}_k^T (\sum_{j \neq k} \mu^2 A_j^2 \mathbf{s}_j \mathbf{s}_j^T + \sigma^2 \mathbf{I})^{-1} \mathbf{s}_k\right)}} \\ &= \sqrt{\frac{\gamma_T(k)}{\left(\mathbf{s}_k^T \mathbf{Q_1}^{-1} \mathbf{s}_k\right)}}, \end{aligned}$$

and

$$\mu r_k(\mathbf{A}) = \mu \sqrt{\frac{\gamma_T(k)}{\left(\mathbf{s}_k^T (\sum_{j \neq k} A_j^2 \mathbf{s}_j \mathbf{s}_j^T + \sigma^2 \mathbf{I})^{-1} \mathbf{s}_k\right)}}$$
$$= \sqrt{\frac{\gamma_T(k)}{\left(\mathbf{s}_k^T (\sum_{j \neq k} \mu^2 A_j^2 \mathbf{s}_j \mathbf{s}_j^T + \mu^2 \sigma^2 \mathbf{I})^{-1} \mathbf{s}_k\right)}}$$
$$= \sqrt{\frac{\gamma_T(k)}{\left(\mathbf{s}_k^T \mathbf{Q}_2^{-1} \mathbf{s}_k\right)}}.$$

Clearly,  $Q_1$  and  $Q_2$  are both real, symmetric and positive definite matrices. Furthermore,

$$\mathbf{Q_2} - \mathbf{Q_1} = (\mu^2 - 1)\sigma^2 \mathbf{I},$$

and since  $\mu > 1$ ,  $\mathbf{Q_2} - \mathbf{Q_1}$  is positive definite. Then, according to Proposition 4.1,  $\mathbf{Q_1}^{-1} - \mathbf{Q_2}^{-1}$  is also positive definite. Hence,

$$\mathbf{s}_{k}^{T}(\mathbf{Q_{1}}^{-1} - \mathbf{Q_{2}}^{-1})\mathbf{s}_{k} > 0$$
  
$$\Leftrightarrow \mathbf{s}_{k}^{T}\mathbf{Q_{1}}^{-1}\mathbf{s}_{k} - \mathbf{s}_{k}^{T}\mathbf{Q_{2}}^{-1}\mathbf{s}_{k} > 0$$

Thus,

$$\mu r_k(\mathbf{A}) = \sqrt{\frac{\gamma_T(k)}{(\mathbf{s}_k^T \mathbf{Q}_2^{-1} \mathbf{s}_k)}}$$
$$> \sqrt{\frac{\gamma_T(k)}{(\mathbf{s}_k^T \mathbf{Q}_1^{-1} \mathbf{s}_k)}}$$
$$= r_k(\mu \mathbf{A}).$$

Thus, the TSG with the LMMSE receiver is shown to have a unique Nash Equilibrium. By using the round robin best response algorithm, the game converges to this Nash equilibrium.

### V. SIMULATION RESULTS

We compare the performance of the MF and LMMSE receivers in a network with 4 secondary users. Every user can adapt its received signal amplitude in the region  $\bar{A}_k = [1, 10]$ mW. In Figs. 2 and 3,  $\rho = 0.1$  and we have used dif-



Fig. 2. High SNR case

ferent noise levels in the simulation. These simulation results show that with the LMMSE receiver, the game converges for all users faster than that with MF under the best response algorithm especially for high SNR values. Here, only the user 1's convergence performance is plotted, since user 1 is the last user that converges its action to the NE ( i.e. the game converges only when the user 1's action converges).



Fig. 3. Low SNR case



Fig. 4. The Transmit Powers at the NE

Figure 4 shows that when achieving the same target SINR, LMMSE receiver consumes less transmit power compared to that of the MF receiver, which is an expected result due to the superior interference suppression capability of the LMMSE receiver.

In Fig. 5, we have shown the convergence performance with different  $\rho$  values for the same target SINR of  $\gamma_T^{MF}(k) = \gamma_T^{MMSE}(k) = 16$ . This comparison shows that with high user cross-correlation, the TSG with the LMMSE receiver can achieve the required target SINR. However, the TSG with the MF receiver can perform poorly for high  $\rho$  values.

### VI. CONCLUSIONS

In this paper, we generalized the target SINR game that is previously defined with a MF receiver to a LMMSE receiver. Our theoretical derivations and numerical results show that there is a unique NE in the TSG with the LMMSE receiver.



Fig. 5.  $\rho = 0.1$  vs  $\rho = 0.8$ 

When achieving the same target SINR, the LMMSE receiver consumes less transmit power than that of MF at the NE, which is expected. More importantly, our results show that convergence time with the LMMSE receiver is less than that with the MF receiver, and the effect is more profound in the high SINR region.

#### REFERENCES

- D. J. Goodman and N. B. Mandayam, "Power control for wireless data," *IEEE Personal Commun.*, vol. 7, pp. 48–54, April 2000.
- [2] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, Feb. 2002.
- [3] F. Meshkati, H. V. Poor, S. C. Schwartz, and N. B. Mandayam, "An energy-efficient approach to power control and receiver design in wireless data networks," *IEEE Trans. Commun.*, vol. 53, no. 11, Nov. 2005.
- [4] F. Meshkati, A. J. Goldsmith, H. V. Poor, and S. C. Schwartz, "A gametheoretic approach to energy-efficient modualtion in CDMA networks with dealy QoS constraints," *IEEE Trans. Commun.*, vol. 25, no. 6, Aug. 2007.
- [5] F. Meshkati, H. V. Poor, and S. C. Schwartz, "Energy-efficient resource allocation in wireless networks: An overview of game-theoretic approaches," *IEEE Signal Proc. Magazine*, pp. 58–68, May 2007.
- [6] N. Gatsis, A. G. Marques, and G. B. Giannakis, "Utility-based power control for peer-to-peer cognitive radio networks with heterogeneous QoS constraints," in *IEEE International Conference on Acoustics*, *Speech and Signal Processing*, 2008. (ICASSP 2008), April 2008, pp. 2805 – 2808.
- [7] W. Wang, T. Peng, and W. Wang, "Optimal power control under interference temperature constraints in cognitive radio network," in *IEEE Wireless Communications and Networking Conference, 2007. (WCNC 2007)*, March 2007.
- [8] M. Hong, H. K. J. Kim, and Y. Shin; "An adaptive transmission scheme for cognitive radio systems based on interference temperature model," in *IEEE Consumer Communications and Networking Conference*, 2008. (CCNC 2008), Jan. 2008, pp. 69 – 73.
- [9] J. O'Daniell, "Analysis and design of cognitive radio networks and distributed radio resource management algorithms," Ph.D. dissertation, Virginia Tech., 2006.
- [10] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, Sep. 1995.
- [11] S. Verdo, Multiuser Detection. Cambridge University Press, 1998.